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MODELING GROWTH STOCKS (PART II)

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ABSTRACT

Continuing the previous work on growth stocks, we propose a diffusion model for growth stocks. Since growth stocks tend to have low or even negative earnings and high volatility, it is a great challenge to derive a meaningful mathematical model within the traditional valuation framework. The diffusion model not only has economic interpretations for its parameters, but also leads to some interesting economic insight — the model postulates mean reversion (with a high mean reverting level) for growth stocks, which could be useful in understanding the recent boom and burst of the "internet bubble". Simulation and an empirical evaluation of the model based on the size distribution are also presented. The simualtion and numerical results are quite encouraging.

1 INTRODUCTION

Although the components of growth stocks may vary over time (perhaps consisting of railroad and utility stocks in the early 1900's, and biotechnology and internet stocks in 2002), studying their general properties is essential to understand financial markets and economic growth in the past, at present, and perhaps in the future too.

Motivated by Ijiri and Simon (1977) on size distribution, Kou and Kou (2001) proposed a discrete model for growth stocks (e.g., biotechnology and internet stocks in 2000 and utility and railroad stocks in the early 1900's). The model only uses a unique feature of growth stocks — their high volatility. Neither earnings (which are not available for most of growth stocks) nor forecasted sales numbers (which are not only unreliable, as evident in the event of the recent "internet bubble", but also lack a clear mathematical relationship with stock prices) are used in the model.

In particular, it is shown in Kou and Kou (2001) that if the market capitalization of the stocks is modeled as a birth-death process, then, in the steady state, the model leads to an almost linear curve for stocks with high volatility (such as biotechnology and internet stocks) when the log-market-capitalization is plotted against the log-ranks; meanwhile for non-growth stocks such a phenomenon should not be expected, primarily because of the very slow convergence of the birth-death process to its steady state distribution due to a low volatility. Therefore, the discrete model also shed light on an empirically observed puzzle that there is an "almost" linear relationship between the logarithm of the market capitalization of growth stocks and the logarithm of their associated ranks, which was first reported in the Wall Street Journal (Dec. 27, 1999) only for internet stocks (this observation was summarized later in a report by Mauboussin and Schay, 2000). Translating into a probabilistic language, this empirical puzzle means that the size distribution of the growth stocks almost follows a power law, and it is not so for ordinary stocks.

This article furthers the study of growth stocks, attempting to find a continuous diffusion model for growth stocks. We achieve it by first consider the weak convergence of the birth-death processes; then, guided by that limit, we investigate a general class of diffusion processes to identify the processes that can lead to the size distribution observed for growth stocks.

The continuous diffusion model also leads to some interesting economic insight. Not only do the parameters in the model have some economic interpretations (see, e.g., Remark 2 in Section 6), but also the model postulates mean reversion for growth stocks with a *high* mean reverting level. This may be useful in understanding the *recent boom and burst of the "internet bubbles"*.

2 THE MODEL

2.1 Review of the Discrete Model

The birth-death process used in Kou and Kou (2001) is a linear birth-death process with immigration and emigration. More precisely, consider at time t a stock with total market capitalization $X(t)$, taking values in non-negative integers $X(t) = i$, $i = 0, 1, 2, ...$ (The unit of $X(t)$ could be, for example, millions or billions of dollars.) The model postulates that given $X(t)$ being in state i, the instantaneous changes are: $i \to i+1$, with rate $i\lambda + g$, $i \geq 0$; $i \to i-1$, with rate $i\mu + h$, $i \geq 1$. The two parameters λ and μ represent the instantaneous appreciation and depreciation rates of $X(t)$ due to the market fluctuation; the model assumes that they influence the market capitalization proportionally to the current value. In general, because of the difficulty of predicting the instantaneous upward and downward price movements, for both growth stocks and non-growth stocks λ and μ must be quite close, $\lambda/\mu \approx 1$. In addition, for growth stocks, both λ and μ must be large, because of the high volatility. The requirement $\lambda < \mu$ is also postulated to ensure that the birth-death process has a steady state distribution. The parameter g models the rate of increase in $X(t)$ due to non-market factors, such as the effect of additional shares being issued through public offerings, or the effect of warranties on the stock being exercised (resulting in new shares being issued). The parameter h models the rate of decrease in $X(t)$ due to non-market factors, such as the dividend payment; for most of growth stocks $h \approx 0$, as no dividends are paid.

2.2 Weak Convergence

In this paper we want to derive continuous diffusion models for growth stocks. An intuitive approach is to consider the limit of the discrete model, with the jump size being 1 and the infinitesimal increment dX_t satisfying

$$dX(t) = \begin{cases} 1, & \text{with prob. } (\lambda X(t) + g)dt + o(dt) \\ -1, & \text{with prob. } (\mu X(t) + h)dt + o(dt) \\ 0, & \text{otherwise.} \end{cases}$$

Now if we let the jump size be Δs, then we have a birth-death process with

$$dX(t) = \begin{cases} \Delta s, & \text{with prob. } (\lambda X(t) + g)dt + o(dt) \\ -\Delta s, & \text{with prob. } (\mu X(t) + h)dt + o(dt) \\ 0, & \text{otherwise.} \end{cases}$$

The mean of $dX(t)$ is $[(\lambda-\mu)X(t)+(g-h)]\Delta s \cdot dt + o(dt)$; the variance of $dX(t)$ is $[(\lambda+\mu)X_t+(g+h)]\Delta s^2 \cdot dt + o(dt)$. If we let $\Delta s \to 0$ in such a way that $(\lambda - \mu)\Delta s \to -\varepsilon\sigma^2 < 0$, $(g-h)\Delta s \to a\sigma^2 > 0$, $(\lambda+\mu)\Delta s^2 \to \sigma^2$, and $(g+h)\Delta s^2 \to 0$, then the limiting stochastic process satisfies

$$E(dX(t)|\mathcal{F}_t) = (-\varepsilon\sigma^2 X(t) + a\sigma^2)dt,$$

$$\text{var}(dX(t)|\mathcal{F}_t) = \sigma^2 X(t)dt.$$

With the above parameterization, it is intuitively reasonable to expect that the birth-death process would converge to a limiting diffusion process that satisfies the stochastic differential equation

$$dX(t) = (-\varepsilon\sigma^2 X(t) + a\sigma^2)dt + \sigma\sqrt{X(t)}dW(t), \quad (1)$$

with $X(0) = x > 0$, where $W(t)$ is a standard Brownian motion. The following proposition makes the intuition rigorous.

Proposition 1. Under suitable regularity conditions, the birth-death process converges weakly to the diffusion process $X(t)$ in (1).

See Kou and Kou (2002) for a list of the regularity conditions, the precise meaning of "converges weakly", as well as the proof of the proposition.

The Model. Proposition 1 motivates us to consider a general model

$$dX(t) = (-\varepsilon\sigma^2 X(t) + a\sigma^2)dt + \sigma X^\gamma(t)dW(t), \quad (2)$$

with $X(0) = x > 0$, where $\gamma > 0$. Note that when $\gamma = 1/2$, we have the limiting diffusion process in Proposition 1.

Of course, without the previous model based on the birth-death process, it would be very hard to imagine a model like (2). Therefore, the discrete birth-death process provides a nice intuition for the continuous diffusion models. However, the diffusion model has its own merits: (1) Generally speaking, diffusion models can lead to many closed form solutions, whence have better analytical tractability than birth-death processes; (2) it is possible to do riskless hedging for diffusion models, while it is impossible to do so for many discrete models.

2.3 High Mean Reverting Level

Before we analyze the model, we must answer a question first: whether the model (2) makes any economic sense.

The most interesting feature of the model (2) is that it has mean reversion which is somewhat unusual for models of stock prices. However, if we assume that $\varepsilon \approx 0$, then the mean reverting level a/ε is *high*. The high mean reverting level effectively yields that, although ultimately the process is mean reverting (in both transient and steady states), within a reasonable time period it may be *very difficult* to observe the mean reverting phenomenon (even in the steady state). Once we accept this model with a high mean reverting level, then it is easy to see that the high mean reverting also provides some interesting insight about *the recent boom and burst of the "internet bubble"*: in other words "what goes up must comes down eventually", but it may take a while to do so (even if the process is already in the steady state). Note the difference between mean reversion and convergence from the transient states to the steady state.

The bottom line is that growth stocks and non-growth stocks may be two different animals, and one may have

mean reverting and the other may not. Therefore, we shall take this assumption as a key feature of the model:
Assumption: $\varepsilon \approx 0$.

Note that, due to the problem of measurement units, it makes more sense to talk about the relative magnitude of ε. In other words, $\varepsilon \approx 0$ means that ε is small relative to other parameters such as a and σ^2.

From a finance viewpoint, this assumption also makes the current model different from the CEV model in Cox and Ross (1976) and CIR model (or the Feller process) in Cox, Ingersoll, and Ross (1985). To be more precise, (1) our model has mean reversion while the CEV model does not. (2) When $\gamma = 1/2$, the same process is called CIR model for interest rates, but there are some major differences — for example, the mean reverting levels are quite different. More importantly, here we point out that the same process with a small negative drift (ε is a very small number) and a high mean reverting level can be used to model growth stocks, giving an explanation to the size distribution puzzle (see Section 5) as well as modeling the boom and burst of the growth stocks.

The current model has several attractive features. (1) It leads to an explanation of the size distribution puzzle, as will be outlined in Section 4. (2) Because the current diffusion model has a simple form, it leads to an analytical expression of the steady state distribution which in turn yields a simple way to price growth stocks relative to their peers; see Sections 3 and 5.2. (3) Recent events related to the boom and burst of the "internet bubble" further indicate the usefulness of introducing the concept of high-level mean reverting to the analysis of growth stocks.

3 PROPERTIES OF THE DIFFUSION MODEL

In general, unless $\gamma = 1/2$ (called Feller process) or $\gamma = 1$ (called Wong process) it is impossible to write down the transition density of $X(t)$ in (2) explicitly. However, we can compute the steady state distribution of $X(t)$ as the following.

Theorem 1. As $t \to \infty$, the distribution of the solution $X(t)$ of (2) converges to a steady state distribution.

(a) When $\gamma > 1$, $f(x)$, the density of the steady state distribution, is given by

$$f(x) = C_2 x^{-2\gamma} \cdot \exp\{2(\frac{\varepsilon}{2\gamma - 2} x^{2-2\gamma} - \frac{a}{2\gamma - 1} x^{1-2\gamma})\}.$$

The tail probability has an asymptotic expression $F(z) := P(X(\infty) > z) \cong C z^{1-2\gamma}$, as $z \to \infty$, where C_2 and C are two normalizing constants. Here and hereafter $a \cong b$ means that $\lim a/b = 1$.

(b) When $\gamma = 1$, $f(x) = C_2 x^{-2(1+\varepsilon)} e^{-2a/x}$, with the tail probability $F(z) \cong C z^{-1-2\varepsilon}$, for some constants C_2 and C.

(c) When $1/2 < \gamma < 1$,

$$f(x) = C_2 x^{-2\gamma} \exp(-\frac{\varepsilon}{1-\gamma} x^{2-2\gamma}) \cdot \exp(-\frac{2a}{2\gamma - 1} x^{1-2\gamma}),$$

with the tail probability $F(z) \cong C z^{-1} \exp(\frac{\varepsilon}{\gamma - 1} z^{2-2\gamma})$.

(d) When $\gamma = 1/2$, $f(x) = C_2 e^{-2\varepsilon x} x^{2a-1}$, with the tail probability $F(z) \cong C e^{-2\varepsilon z} z^{2a-1}$, for some constants C_2 and C.

(e) When $0 < \gamma < 1/2$,

$$f(x) = C_2 x^{-2\gamma} \exp(-\frac{\varepsilon}{1-\gamma} x^{2-2\gamma}) \cdot \exp(-\frac{2a}{2\gamma - 1} x^{1-2\gamma}),$$

and

$$F(z) \cong C z^{-1} \exp(-\frac{\varepsilon}{1-\gamma} z^{2-2\gamma}) \cdot \exp(-\frac{2a}{2\gamma - 1} z^{1-2\gamma}),$$

for some constants C_2 and C.

Here we only show Case (d), which is the easiest one to prove. The proofs for the other cases are more complicated; see Kou and Kou (2002). For $\gamma = \frac{1}{2}$, $X(t)$ is the well-known square-root process (see Karlin and Taylor, 1981, p. 334), whose steady-state distribution is well known to be gamma with density $f(x) = C e^{-2\varepsilon x} x^{2a-1}$. It follows easily that the tail probability $F(z) \cong C e^{-2\varepsilon z} z^{2a-1}$.

4 GENERAL PROPERTIES OF SIZE DISTRIBUTION

Consider M (here M is an unknown quantity) growth stocks governed by the same diffusion process (2), among which the K largest stocks (in terms of their market capitalization) are included in a group to be studied. Suppose we rank the market capitalization from 1 to K and denote the resulting ranked values as $X_{(1)}$, $X_{(2)}$, ..., $X_{(K)}$, with $X_{(1)}$ being the largest, and $X_{(2)}$ the second largest etc. Then the empirical tail distribution $\tilde{F}(x)$ (the empirical version of F) evaluated at $X_{(i)}$ is simply $\tilde{F}(X_{(i)}) = i/M$, $i = 1, ..., K$. Now assume

(A1) The diffusion process has reached the steady state.

(A2) For each stock included in the group, the market capitalization is large; in other words, even $X_{(K)}$ is large.

Then we can apply the result of Theorem 1 to study the size distribution of growth stocks. It is worth pointing out that assumption (A2) implies that the model is only valid for large-cap growth stocks. According to Theorem 1, there are five different cases for the size distribution. Table 1 below summarizes the size distribution under the five cases.

Table 1: Size Distribution in Five Cases.

Cases	Slope in the size distribution:
$\gamma > 1$	$-\frac{1}{2\gamma-1}$
$\gamma = 1$	$-\frac{1}{1+2\varepsilon}$
$\frac{1}{2} < \gamma < 1$	-1
$\gamma = \frac{1}{2}$	$-\frac{1}{1-2a}$
$0 < \gamma < \frac{1}{2}$	Nonlinear pattern

The detailed derivation of the above result is given in Kou and Kou (2002). Here we only give a derivation of Case (d), $\gamma = 1/2$, as it is the simplest one. By Theorem 1, in the steady state, for large z, $\log F(z) \approx -2\varepsilon z - (1-2a)\log z + C$, for some constant C. Therefore, empirically with $X_{(i)} = z$, we shall expect that $\log \tilde{F}(X_{(i)}) = \log(i/M) \approx -2\varepsilon X_{(i)} - (1-2a)\log X_{(i)} + C$. Rearranging the terms above yields

$$\log X_{(i)} \approx C_M - \frac{1}{1-2a}\log i - \frac{2\varepsilon}{1-2a} X_{(i)} \qquad (3)$$

for some constant C_M that depends on M. Hence, the slope in the size distribution is $-\frac{1}{1-2a}$.

Equation (3) provides a link between the market capitalization of the stocks and their relative ranks within the group. However, since it involves a nuisance parameter C_M, a better equation can be obtained by eliminating C_M first, as is typical in many standard statistical procedures. This can be done by taking the difference of $\log X_{(i)} - \log X_{(1)}$: for $1 \le i \le K$,

$$\log \frac{X_{(i)}}{X_{(1)}} \approx -\frac{1}{1-2a}\log i - \frac{2\varepsilon}{1-2a}\left(X_{(i)} - X_{(1)}\right). \qquad (4)$$

Thanks to the assumption $\varepsilon \approx 0$, the last term in (4) is generally negligible.

5 SIZE DISTRIBUTION FOR BIOTECHNOLOGY AND INTERNET STOCKS

5.1 Explaining the Size Distribution Puzzle

For biotechnology and internet stocks, the empirical evidences suggest that the slope of the size distribution is always less than -1. Therefore, in view of the result of the previous section, for biotech and internet stocks γ must be $1/2$ in the model (2). In other words,

$$dX(t) = (-\varepsilon\sigma^2 X(t) + a\sigma^2)dt + \sigma\sqrt{X(t)}dW(t), \qquad (5)$$

with $X(0) = x > 0$, which corresponds to the Feller process also used in finance as the CIR model for the spot interest rate (but here we have a high mean reverting level).

Remark 1. This, however, does not imply that for other growth stocks, such as railroad and utility stocks back in the 1900's, or for any new groups of growth stocks in the future, γ must be $1/2$. It only says that currently for biotechnology and internet stocks γ appears to be $1/2$.

Now recall for large growth stocks (thus satisfying assumption A2) we have derived in the previous section the steady state size distribution (4), which explains why a plot of log-market-capitalization versus log-rank displays a linear pattern. However, it is important to note that the size distribution in steady state is only relevant if the convergence from the transient states to the steady state is fast enough, i.e. if the convergence can be observed in a timely fashion. A good measure of the convergence speed is the decay parameter defined by

$$\delta := \sup\{\alpha \ge 0 : p(t, x, y) - p(y) = O(e^{-\alpha t}), \forall x > 0\},$$

where $p(t, x, y) = \mathsf{P}(X(t) \in dy | X(0) = x)$ is the transition density of $X(t)$ and $p(y)$ is the steady-state density function. Immediately from an expansion in Karlin and Taylor (1981, p. 334), we get $\delta = \varepsilon\sigma^2$.

Two comments are needed. First, it is well known that, due to the problem of measurement units, it is better to compare the relative magnitude of different δ's, rather than focusing on the absolute magnitude of δ, which may not provide much information. More precisely, comparing decay parameters may give us some idea of different convergence speeds among various stochastic processes. Second, the decay parameter δ affects the convergence *in an exponential way*; in other words, a small difference in δ can have a remarkable effect on the speed of convergence.

This helps to explain why the almost linear relationship between the logarithm of the market capitalization and the logarithm of the ranks does not appear for non-growth stocks. There are at least two reasons. First, the mean reverting diffusion model may not be valid for non-growth stocks. Second, even if the model is valid for non-growth stocks, in order to empirically observe such a linear phenomenon as implied by (4), the convergence from the transient states to the steady state must be fast enough. This in turn depends on the magnitude of the decay parameter δ.

It is well known the volatility for growth stocks is much larger than that of the non-growth stocks. For example, Kerins, Smith, and Smith (2001) show empirically that the volatility of internet stocks may be at least five times that of traditional stocks. In the model (5), if σ of growth stocks is five times larger, then σ^2 is 25 times larger! This leads to in a much larger decay rate δ (which affects the convergence in an exponential way). Therefore, for non-growth stocks, although in the steady state plotting the logarithm of the market capitalization against the logarithm of the relative ranks might display a linear relationship, the linear relationship may not emerge at all within a reasonable amount of time, due to the slow convergence from the transient states to the steady state.

5.2 Relative Pricing of Growth Stocks

Equation (4) provides a way to price a growth stock relative to its peers within the group (the contribution of the peer group is to provide an estimate of a and ε, and the relative ranks) by running a nonlinear regression subject to the constraints $a > 0$ and $\varepsilon > 0$. Once these parameters are obtained, the theoretical market capitalization of the stock can be calculated according to equation (4), with the input being its rank.

To use the model to relatively price large-cap growth stocks, it is important to keep in mind that the stocks within the peer group should have similar parameters a and ε (for example, it may not be sensible to group biotechnology stocks with internet stocks as their parameters may be quite different). However, in principle, the relative pricing does not require σ^2 to be the same; the only requirement is that σ^2 must be very large, as σ^2 only controls the speed of convergence from transient to steady state and does not enter the equation (4).

6 SIMULATION AND NUMERICAL ILLUSTRATION

To this whether the model (1), fits our intuition of growth stocks, Figure 1 provides an illustration of the model by simulating a sample pat of (1) for about 10 years. The parameters used here are: $X(0) = 500$, $\varepsilon\sigma^2 = 0.001$, $a\sigma^2 = 0.5$, and $\sigma = 100\%$.

The sample path suggests two things: (1) The sample path is quite volatile. (2) Although the sample path may have a mean reversion, one may not notice the mean reversion even within 10 years, which confirms the theoretical property of the model that mean version may be difficult to be observed. Note the difference between mean reversion (in both transient and steady state) and convergence to the steady state.

To illustrate the results of the size distribution, for biotechnology stocks we plot the logarithm of their market capitalization relative to the largest biotechnology stock versus the logarithm of their ranks, i.e. $\log(X_{(i)}/X_{(1)})$ versus $\log i$. The list of 139 biotech stocks is given in Appendix C in Kou and Kou (2001). The result indicates

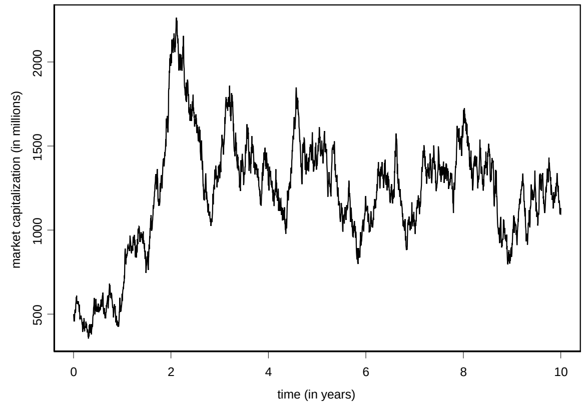

Figure 1: A Simulated Sample Path of the Model

that it shows a clear linear trend, a pattern also predicted by the diffusion model. In contrast, for other non-growth stocks with low volatility, such as Dow transportation and saving and loan stocks, the plot suggests that the pattern of the size distribution is far from linear, which is again expected from the model here.

Table 2 reports the estimated $\hat{a}$ and $\hat{\varepsilon}$ from (4), as well as the R^2 for six trading days, which represent days from January 2, 1998, and every 100 trading days onward. Note that, comparing to $\hat{a}$, the estimated $\hat{\varepsilon}$'s are all very small, confirming our earlier assumption $\varepsilon \approx 0$. The R^2 being at least 97% directly supports the prediction of the model. The regression results of internet stocks are quite similar to those of biotechnology stocks, and are omitted here. Table 3 reports the estimated parameters and the R^2, as of August 22, 2001 (after the burst of the "internet bubble"). Again the R^2 is at least 96%. The fitting is good even under this severe market downturn.

Table 2: Estimated a and ε for Biotechnology Stocks

	$\hat{a}$	$\hat{\varepsilon}$	R^2
Jan 2, 98	0.0400	6.905×10^{-10}	97.8%
Aug 7, 98	0.0825	6.260×10^{-10}	98.2%
Mar 15, 99	0.1475	5.285×10^{-10}	98.3%
Oct 15, 99	0.1360	5.460×10^{-10}	99.2%
May 19, 00	0.0985	6.020×10^{-10}	98.6%
Dec 21, 00	0.1325	2.826×10^{-9}	97.5%

Table 3: The R^2 and Estimated Parameters for the Recent Market (August 22, 2001)

	$\hat{a}$	$\hat{\varepsilon}$	R^2
Biotech Stocks	0.096	4.615×10^{-7}	96.4%
Internet Stocks	0.181	1.715×10^{-6}	98.5%

Remark 2. As we mentioned before, the parameter a attempts to measure the magnitude of money inflow to the stock due to non-market factors, such as exercising of employee stock options and public offering of new/additional shares, etc. It is interesting to see that in the above numerical

examples, a tends to be bigger between March 1999 and October 1999 (when the activities of public offerings were quite frequent), and again around December 2000 (when many employees began to exercise their stock options at the beginning of the current bear market).

7 DISCUSSION

Under the model (2), it is ready to derive option pricing formulae. This is because under the risk neutral measure $\tilde{P}$, the dynamics becomes the CEV process; thus, one can simply use the results in Cox and Ross (1976) for call and put options, and Davydov and Linetsky (2001) for path-dependent options.

The diffusion model proposed in this paper should only be viewed as a understanding of growth stocks, not as a trading tool. This is mainly because the relative pricing formula needs the ranks of market capitalization as the input; but the model does not directly provide a dynamics of the ranks. Nevertheless, hopefully the results may stimulate some further discussions of using some non-standard methods, such as size distribution, to analyze a non-standard (yet important) finance problem.

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