Applied Mathematics 201
Physical Mathematics I

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Meeting time and location: (Fall 2014) Mon and Wed, 1pm-2:30pm in Pierce 209.

Office hours:
Mauricio: Wed, 10:30am-12pm in Pierce 312
Avi: Thurs, 1pm-2:30pm in Pierce 328B
(Please e-mail us to request an appointment outside of the official office hours)

1. Prerequisites:

Applied Mathematics 104 (*Series expansions and complex analysis*) and 105 (*Ordinary and partial differential equations*). Programming experience in MATLAB (or another language) is required.

2. Course description:

The goal of this course is to give a modern introduction to mathematical methods for solving hard mathematics problems that arise in the sciences. The main focus will be to explain the process of applied mathematics, namely how to take a hard problem, of the type ordinarily encountered in applications, and gain insight into its important features. Applied Mathematics is a no-holds-barred competition, in which one uses all available tools to understand a problem as much as possible. The approach requires a combination of (a) "real" mathematics, comprised of theorems and exact results; (b) courage and skill in making legitimate approximations; and (c) intelligent use of computers to both verify and extend the validity of the approximations.

Theory, Approximate techniques, and Numerical methods will be taught as needed to solve the problems at hand. We will discuss these methods in the context of mathematics problems that arise in a variety of fields, ranging from pure mathematics (e.g. the zeros of the Riemann zeta function), to optics (e.g the colors of the rainbow), to quantum mechanics (e.g the semi-classical limit), to fluid mechanics. We will start with simple problems (polynomial equations, simple integrals and simple differential equations) and end the semester with a study of nonlinear partial differential equations. We will try to convince you that one can understand quantitative features of arbitrarily hard mathematics problems, by intelligently combining all of the resources (computational and analytical) that you have at your disposal.

In addition to the material we have constructed a wikipedia style web site for the course. This wiki contains the class notes, and part of the class will involve commenting and making sure the ideas are clear. Instead of the usual mechanism, homework will be carried out on the wiki. Each homework "set" will consist of two stages. In the first stage each person is initially responsible for solving about two problems on their own and putting the solution on the wiki. In the next stage each person is responsible for improving and understanding several solutions others have written.

2.1 Course objectives: At the completion of the course, students will be able to:

- Identify appropriate mathematical techniques to obtain reliable approximate solutions to a wide variety of problems,
- Understand the assumptions, limitations, and validity of such techniques,
- Implement these mathematical techniques in MATLAB.
- Communicate precise mathematical ideas in writing supported by graphics.

3. Required work and grading criteria

You are expected to attend and participate in class.

3.1 Grading: Problem sets 50%, Final project 30%. There will be one take-home mid-term exam (Mid-November) 20%.

3.2 Problem sets: They will be assigned approximately bi-weekly. Most of them will have an analytical and a computational (programming) component. MATLAB will be the programming language of the course; however, students are welcome to use other programming languages based on their previous knowledge. We suggest that people with limited experience using MATLAB visit the web-based tutorial at:

http://www.mathworks.com/academia/student_center/tutorials/register.html

to become familiar with it. Sections early in the class will focus on how to use MATLAB.

Wiki. You will solve the assigned problems with a team of class mates, and your solutions will be submitted (uploaded) using a "wiki" tool online. We will provide support on how to edit the wiki early in the semester. We will create an account at the site for everyone in the course. Editing is actually quite easy and just requires entering text. The only difficulty may be learning how to write mathematical formulas, which requires knowing a little bit of LaTeX. We will teach this to you, and there is excellent documentation provided on the wiki.

3.3 Final project: One of the main goals of the class is for you to use the techniques introduced in the class to actually solve a problem you may be interested in. This will help you realize that what we will study can be very powerful to solve real-life problems. With this in mind, you will work on a final project. Depending on the size of the class you may work with another student. We will help you choose a problem that requires the use of analytical and computational tools of interest to you during the semester. You will work on your project at least during the last several weeks of classes. Mauricio and Avi will be available to discuss your ideas regularly during those weeks and throughout the semester.
You are required to turn in a one-page **project proposal by November 1st, 2014** (this submission will not be graded; it is intended to get you started with your project). A written **final project is due on December 10th**.

4. Topics covered in this class:

- What is an analytic solution? What is an approximate solution?
- Analytical and numerical solutions to polynomial equations (Eigenvalue problems)
- Dimensional Analysis
- Analytical and numerical solutions to ordinary differential equations
- Analytical and numerical integration of real and complex functions.
- Analytical and numerical solutions to partial differential equations

5. Textbook: We will follow the class notes written by Prof. Michael Brenner for this class. They are available at:

http://esag.harvard.edu/rice/AM201_Brenner,Michael_CourseNotes_2010.pdf.

Reference books that you may find interesting for the class include:

- G. F. Carrier, M. Krook and C. E. Pearson, *Functions of a Complex Variable*, McGraw Hill, 1966; reprinted by Hod Books, Ithaca, 1983, and by Society of Industrial and Applied Mathematics (SIAM), 2005, as Classics in Applied Mathematics 49. (6 copies on reserve at McKay Library)
- C. M. Bender and S. A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, McGraw-Hill, 1978, 1999. (3 copies on reserve at McKay)
- E. J. Hinch, *Perturbation Methods*, Cambridge University Press, 1991. (2 copies to be on reserve at McKay, 1 available now)
- R. L. Graham, D. E. Knuth and O. Patashnik, *Concrete Mathematics: A foundation for computer science*, Addison Wesley, 1994. (1 copy to be on reserve at McKay)

Other books of relevance are on reserve in the Gordon McKay library.