

Applied Mathematics 120

Applicable Linear Algebra

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Meeting time and location: Tue and Thu, 2:30pm-4pm at Cruft 309

Office hours: Tue and Thu, 4pm - 5pm in Pierce 312
(Please e-mail me to request an appointment outside of the official office hours)

1. Course description:

Relevant issues in our society such as wireless communications, climate change, internet search engines, and financial strategies are addressed using computational techniques. Linear Algebra is one of the most important and fundamental topics behind these computations. In designing a strategy to solve complex real-life problems, we frequently “linearize” and/or “discretize” the problem at hand and obtain a manageable “linear” approximation. This simplified version of the problem (or approximation) typically involves solving one or a collection of linear problems, either directly or using iterative processes. In this course, we will study how to solve the aforementioned linear problems. This will be done by reviewing some of the material that you learned in Applied Math 21b, and by introducing efficient and accurate computational techniques to solve the actual complex problems with the aid of computers. We will use many applications from computer science, physics, image processing, data processing, climate, chemistry, biology, and finance, to motivate the techniques introduced in the course.

I will strive to maintain a balance between the theory (mathematical foundation) and practice (algorithms implementation using a computer). Many assignments will be a combination of both aspects. The final project will be a hands-on experience solving a real life problem.

We will use matlab as the scientific computing language for the course, as it provides us with a simple interface to implement ideas and graphically visualize them. Also, many well known linear algebra routines have been incorporated in this software allowing us to easily expand them in order to solve a particular problem. Programming support will be offered for matlab if students require it.

1.1 Course objectives:

At the completion of the course, students will be able to:

- identify an appropriate linear algebra approach/algorithm for specific goals/applications,
- understand the requirements, limitations and validity of such algorithms in a given mathematical problem,
- understand the theory and implementation of these algorithms using matlab,
- utilize professionally implemented linear algebra routines/functions/algorithms in order to formulate appropriate strategies to solve challenging and complex questions.

2. Required work and grading criteria

You are expected to attend and participate in class.

2.1 Homeworks: They will contribute to 40% of your grade, and will be assigned approximately weekly. You are expected to turn them in a week after they are assigned. These homeworks will help you become familiar with both the concepts introduced in the class and the matlab programming environment. Grading of homeworks will emphasize your understanding and ability to solve problems using the techniques introduced in the class, and as many homeworks will involve the use of matlab, we will consider three aspects when grading the computational sections: (I) whether the program works, i.e. whether you get the expected results using your code, (II) whether the program is easy to understand: well-structured and well-documented, and (III) whether it could easily be used or adapted to solve similar problems. In other words I will encourage you to write clean, efficient and easy to understand codes.

Late homework policy: We will accept one late homework per student (up to five days late) with a 20 % penalty (provided the solution of the problems have not been discussed in the class or posted online).

2.2 Mid-term and final: They will contribute to 40% of your grade (20 % each). The mid-term exam will be 1 hour and 30 mins, whereas the final will be 3 hours.

2.3 Final project: 20% of your grade will come from a final project. I will help you choose a fun and interesting topic, and help you design a meaningful problem that you can solve using a collection of linear algebra techniques in approximately a month. I will be available to discuss your ideas throughout the whole semester, and especially one week before the “short description” of your project is due, on Nov 4th, 2011. No homework will be assigned for the last two weeks of classes to give you more time to devote to your final project. The final projects need to be approved by me by Nov 11th, and you will have at least a month to work on them.

2.3.1 Short description (due on Nov 4th, 2010): This brief (2 pages approximately) report should contain a description of the problem that you intend to solve as your final project along with a proposal containing the linear algebra strategies that you intend to use in order to solve it. This description will not be graded but you are required to turn it in so that I can approve your project.

2.3.2 Final project report (due on Dec 11th, 2010): This report will contain (I) a motivation stating why this topic/problem is relevant, (II) a well posed mathematical problem to be solved, (III) background of the linear algebra (and its numerical/computational algorithm) approach utilized to solve it, (IV) results, and (V) a discussion of the results and their relevance to the problem at hand. A working matlab program file containing the implemented numerical approach used is expected. A clear and concise presentation of all these aspects will contribute to your grade. Further details on the format and length of this final project will be available on Nov 11th.

You are expected to complete the online Matlab tutorials (2 hours approx.) found at:
http://www.mathworks.com/academia/student_center/tutorials/register.html

Note: if the size of the group is small enough (to be determined after shopping period) students may choose to opt out from the final exam and get 40% of their grade from the final project.

3. Textbook:

There is no required textbook for the class, however, if you wish to buy a text book for the class I would recommend: *Linear Algebra and its Applications* by G. Strang. You may also consider *Linear Algebra. A Modern Introduction* by D. Poole. The two of them are good introductory-level books for the theoretical background of the class. An introductory-level book for the computational algorithms is: *Scientific Computing. An intro-*

ductory survey by M. Heath. During the semester, I will present material from diverse sources that I will make available as new topics are introduced at the request of students.

As for the programming aspects of the class you may find the following references useful: *MATLAB guide* by D. J. Hingham and P. J. Hingham (SIAM, 2005) and the online MATLAB documentation at www.mathworks.com

4. Topics covered in the class:

4.1 Mathematical Foundations:

- Linear systems of equations and its matrix representation. (Geometric interpretation)
- Gaussian elimination and Triangular systems.
- LU factorization: direct methods.
- Vector spaces: The four fundamental spaces.
- Iterative methods.
- Orthogonality and Least squares problems.
- Eigenvectors and Eigenvalues.
- Singular Value Decomposition (SVD).

4.2 Computational Algorithms:

- Computer representation of numbers (Floating point numbers and arithmetics, finite precision, error propagation and catastrophic cancellation).
- LU factorization with finite precision arithmetics (Partial and total pivoting).
- Banded systems (Efficient methods for sparse matrices) .
- Iterative methods to solve huge matrices (Jacobi, Gauss-Seidel, SOR).
- Gram-Schmidt orthogonalization method (QR factorization and Householder transformations).
- Power method to compute eigenvalues and eigenvectors

4.3 Applications:

- Information theory (Google's PageRank algorithm and Multiple Antenna Systems), Global Positioning System, Data Compression and Image Processing, Fast Fourier Transform, Principal Component Analysis and Empirical Orthogonal Function (EOF) Analysis, Leontief Economy models, Stiff Systems in Chemistry.