Water vapor impacts on convective activity

Olivier Pauluis (NYU)

Convection workshop

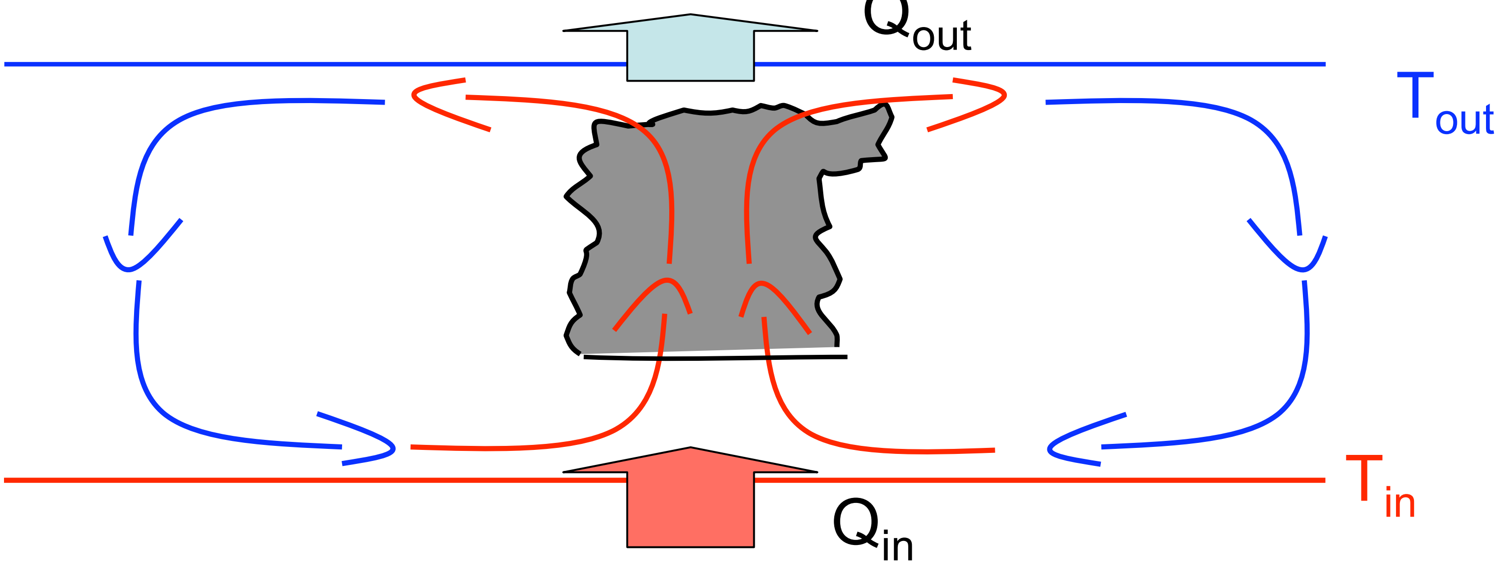
Harvard University

October 17th 2009

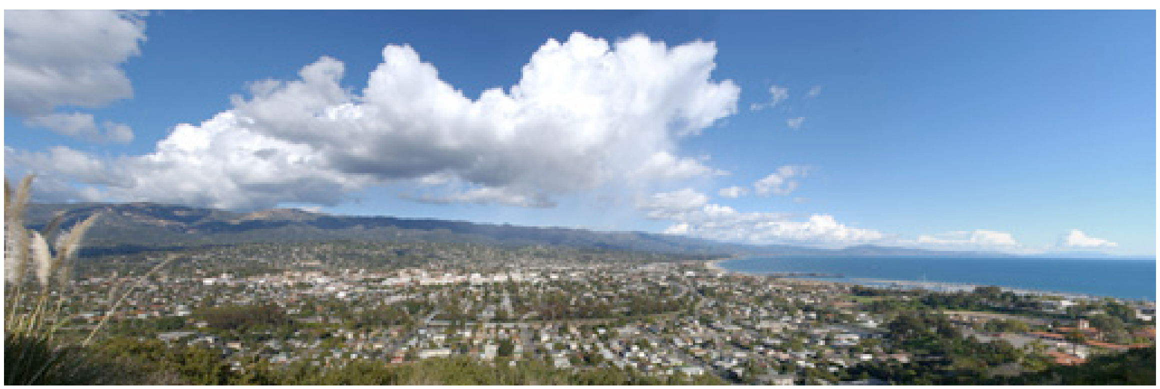
Outline

- 2 paradigms:
  - Convection is associated with an upward buoyancy flux;
  - this upward buoyancy flux itself is tied to an upward energy transport.
- But evidence indicates that atmospheric relative humidity affects convective activity.
- Two key findings:
  - Relative humidity modulates the work done by an idealized thermodynamic cycle;
  - Expression for the buoyancy flux matches the work done by such an idealized cycle.

I. Idealized heat engine



- Idealized problem: convection transport water vapor and energy upward from a warm/moist source to a dry/cold sink.
- Situation is analogous to shallow, non-precipitating convection.



Ia. Carnot cycle

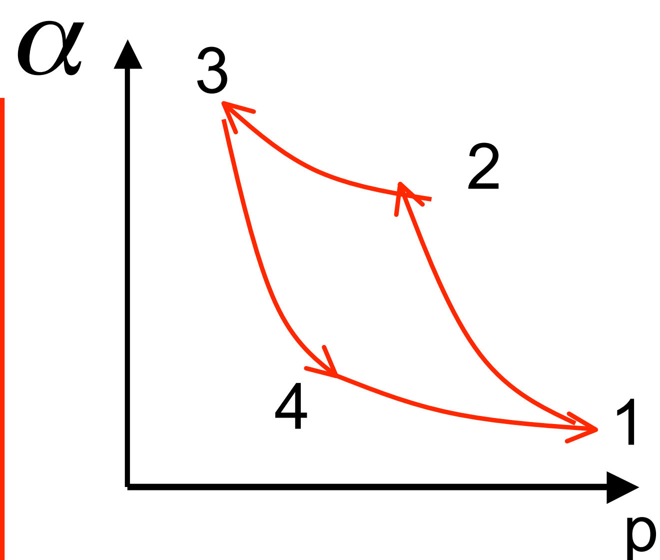
$1 \rightarrow 2$: isothermal expansion at T_{in}

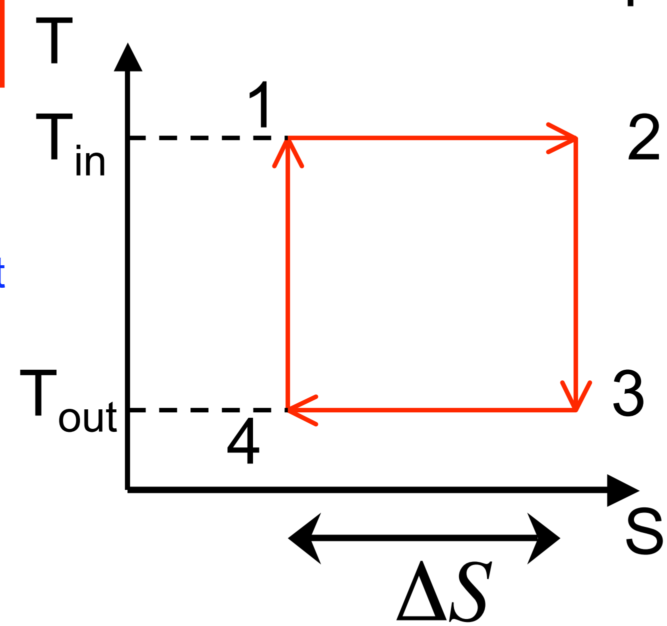
$2 \rightarrow 3$: adiabtic expansion with $S_2 = S_3$

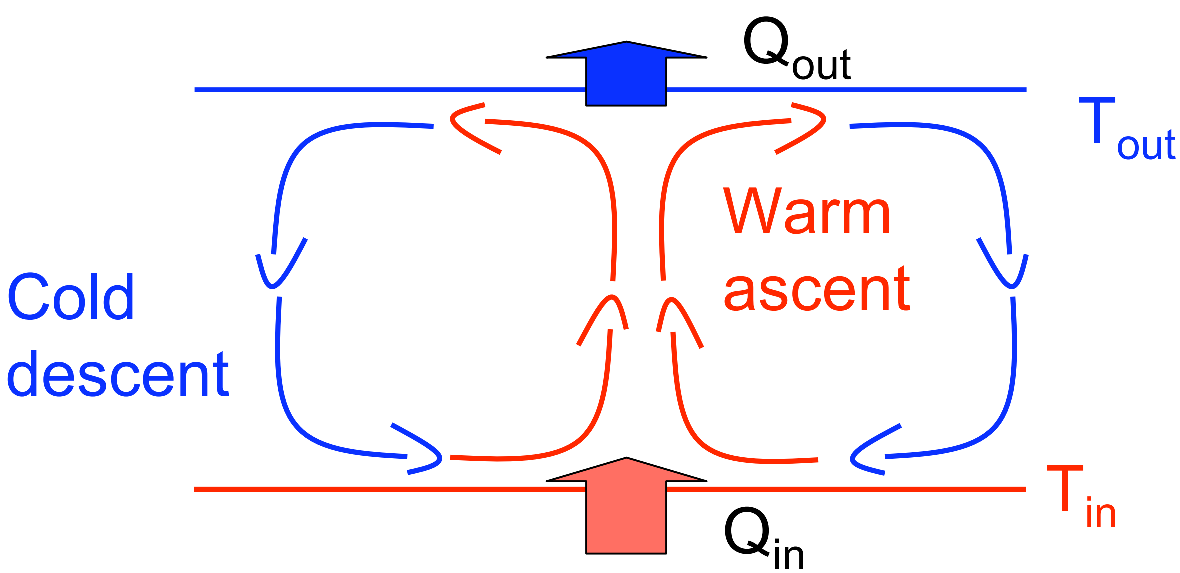
$3 \rightarrow 4$: isothermal compression at T_{out}

$4 \rightarrow 1$: adiabatic compression with $S_2 = S_3$

Total water content is constant through the entire cycle!







- Mechanical work is defined as

$$W = \oint -\alpha(S, r_T, p)dp$$

- Using the thermodynamic relationship

$$TdS = dh - \alpha dp - gdr_T$$

we get:

$$W = \oint TdS + \cancel{\oint gdr_T} = (T_{in} - T_{out})\Delta S$$

$dr_T=0$

- External heating

$$\delta Q = dh - \alpha dp = TdS$$

- Heating at the warm source:

$$Q_{in} = \oint \delta Q^{+} = \int_1^2 TdS = T_{in}\Delta S$$

$$\boxed{\text{Efficieny } \eta_C = \frac{W}{Q_{in}} = \frac{T_{in} - T_{out}}{T_{in}}}$$

Ib. Steam cycle

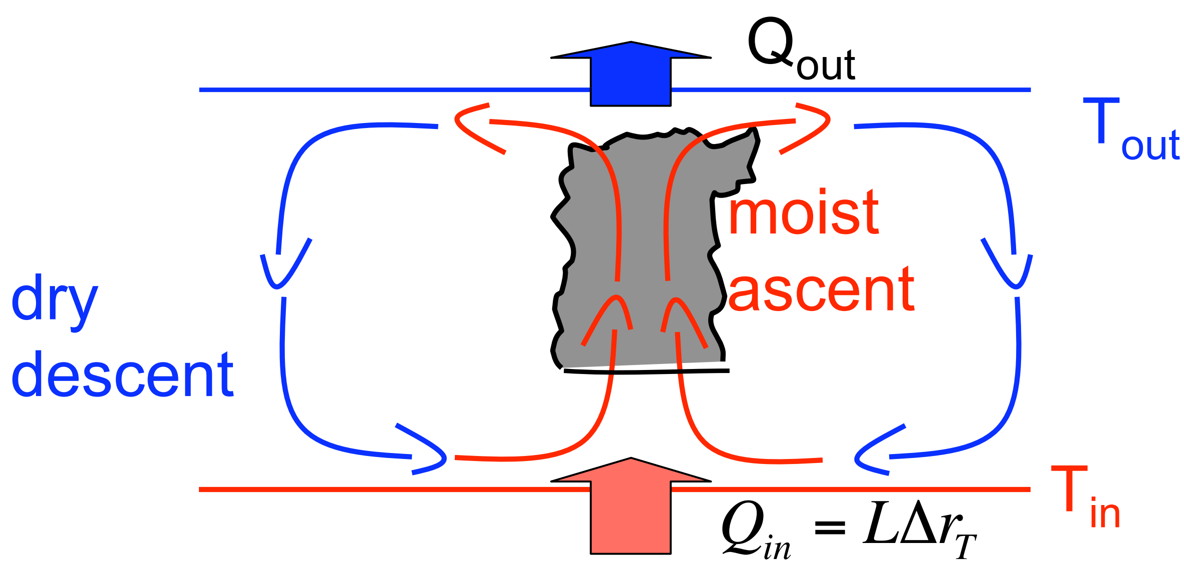
$1 \rightarrow 2$: isothermal moistening at T_{in}

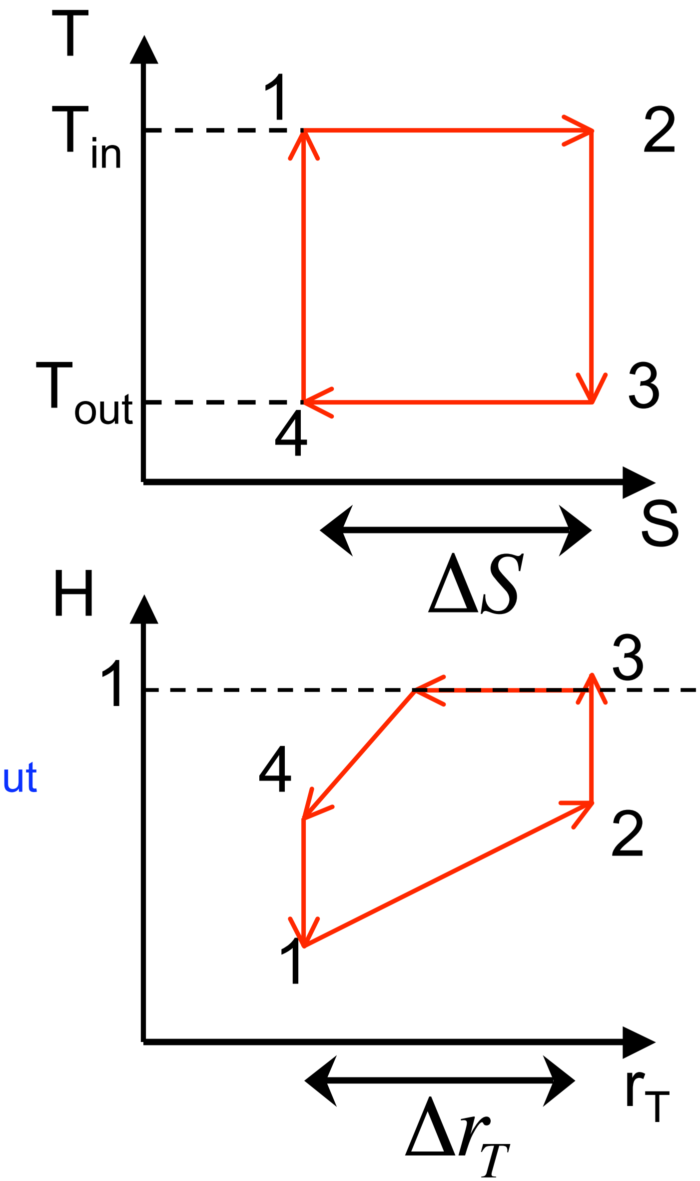
$2 \rightarrow 3$: adiabtic expansion

$3 \rightarrow 4$: isothermal drying at T_{out}

$4 \rightarrow 1$: adiabatic compression

Heating is due solely to evaporation!





- Mechanical work:

$$\boxed{\begin{aligned} W &= \oint T dS + \oint g dr_T \\ &= (T_{in} - T_{out})\Delta S + (g_{in} - g_{out})\Delta r_T \end{aligned}}$$

$g = R_v T \ln H$: Gibbs free energy of water vapor

$$g_{in} = R_v T_{in} \ln H_{in}$$

$$g_{out} = R_v T_{out} \ln H_{out}$$

- Surface heating: $Q_{in} = T_{in}\Delta S + g_{in}\Delta r_T = L\Delta r_T$

- Entropy change: $\Delta S = \left(\dfrac{L - g_{in}}{T_{in}}\right)\Delta r_T$

$$\boxed{\text{Efficieny } \eta_H = \frac{W}{Q_{in}} = \frac{T_{in} - T_{out}}{T_{in}} + \frac{R_v T_{out}}{L} \ln \frac{H_{in}}{H_{out}}}$$

$$\boxed{\text{Efficieny } \eta_H = \underbrace{\frac{T_{in} - T_{out}}{T_{in}}}_{\text{Carnot efficiency}} + \underbrace{\frac{R_v T_{out}}{L} \ln \frac{H_{in}}{H_{out}}}_{\text{additional term depending on saturation}}}$$

- The efficiency depends on the state of the system!!!
- Saturated case: *H=1*

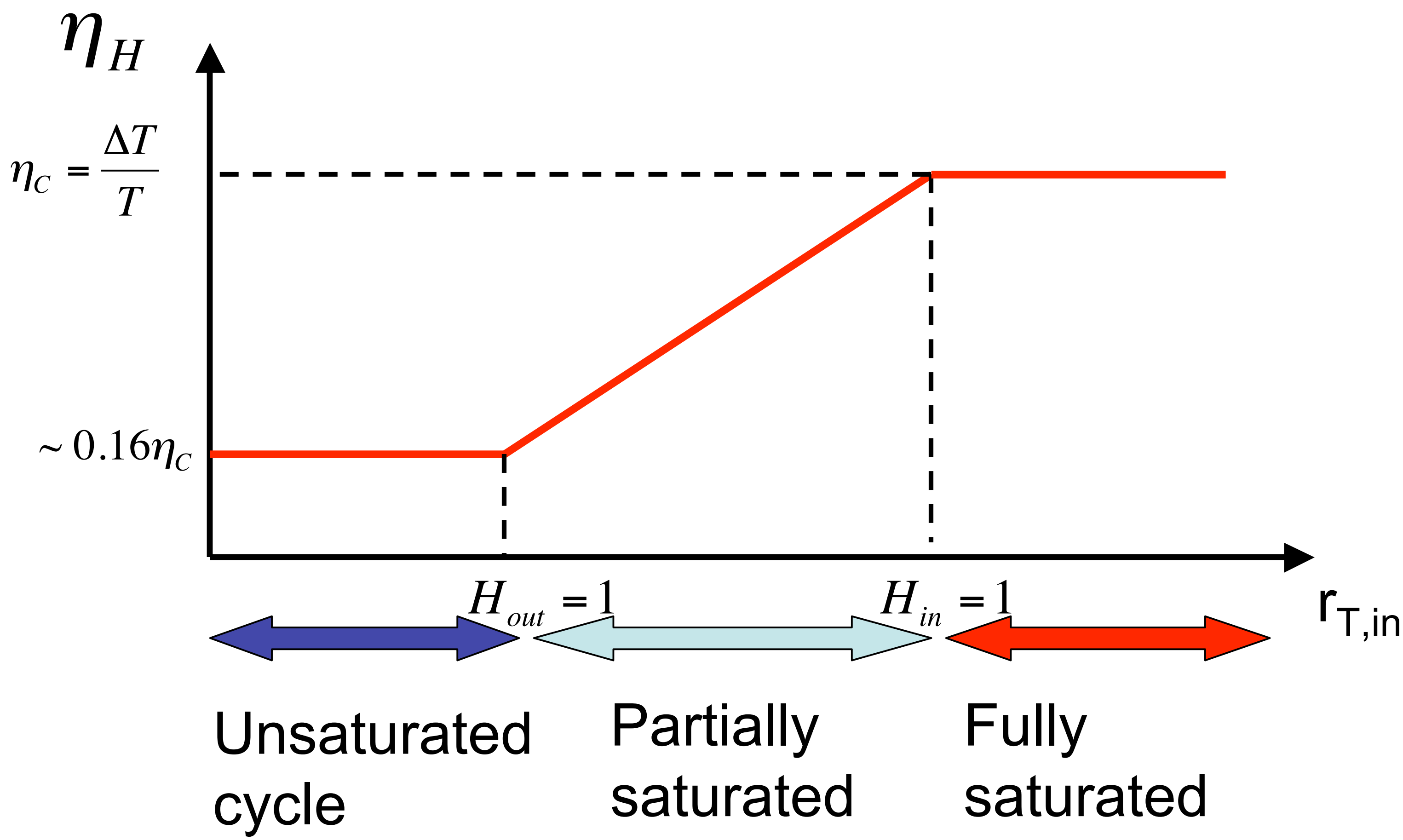
$$\text{Efficieny } \eta_{H,sat} = \frac{W}{Q_{in}} = \frac{T_{in} - T_{out}}{T_{in}}$$

- General case: the relative humidity increases with height, i.e

$$\boxed{H_{out} \geq H_{in} \rightarrow \eta_H \leq \frac{T_{in} - T_{out}}{T_{in}}}$$

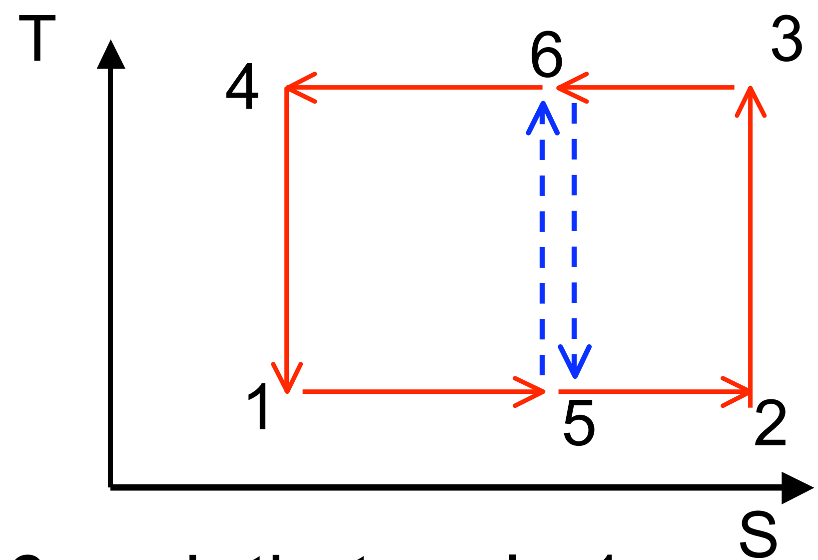
- Hence, a steam cycle produces at best as much mechanical work as a Carnot cycle

- Three regimes:
  - The cycle is unsaturated at all time: efficiency is minimum.
  - The cycle is partially saturated: efficiency increases with amount of water in the cycle.
  - The cycle is saturated at all time: efficiency is maximum and given by the Carnot efficiency



Ic. Mixed steam-Carnot cycle

$1 \rightarrow 2:$ isothermal heating and moistening at T_{in}
$2 \rightarrow 3:$ adiabtic expansion
$3 \rightarrow 4:$ isothermal cooling and drying at T_{out}
$4 \rightarrow 1:$ adiabatic compression



- Intermediary steps 5 and 6 such that cycle 1-5-6-4 is a humidifier and 5-2-3-6 is a Carnot cycle.
- Forced by a combination of sensible and latent heating

II. Buoyancy flux

- We consider now a convective layer. The generation of kinetic energy is given by the vertical integral of the buoyancy flux:

$$\frac{dKE}{dt} = \int \rho_0 \overline{w'B'} dz$$

- B is the buoyancy

$$B(S, r_T, z) = G \frac{\alpha_p - \alpha_0(z)}{\alpha_0(z)}$$

IIa. Dry case:

- We rewrite the buoyancy perturbation as

$$B' = G\frac{T'}{T_o}$$

- The buoyancy flux is then

$$\overline{w'B'} = \frac{G}{C_p T_0} C_p \overline{w'T'} = \frac{\Gamma_d}{T_0}\overline{w'h'}$$

- And the generation of kinetic energy is

$$\frac{dKE}{dt} = \int \rho_0 \overline{w'B'} dz = \frac{\Gamma_d \Delta z}{T_0}\overline{w'h'}$$

$$= \frac{\Delta T}{T_0}\overline{w'h'}$$

IIb. Stratocumulus convection

- Linearize the buoyancy flux

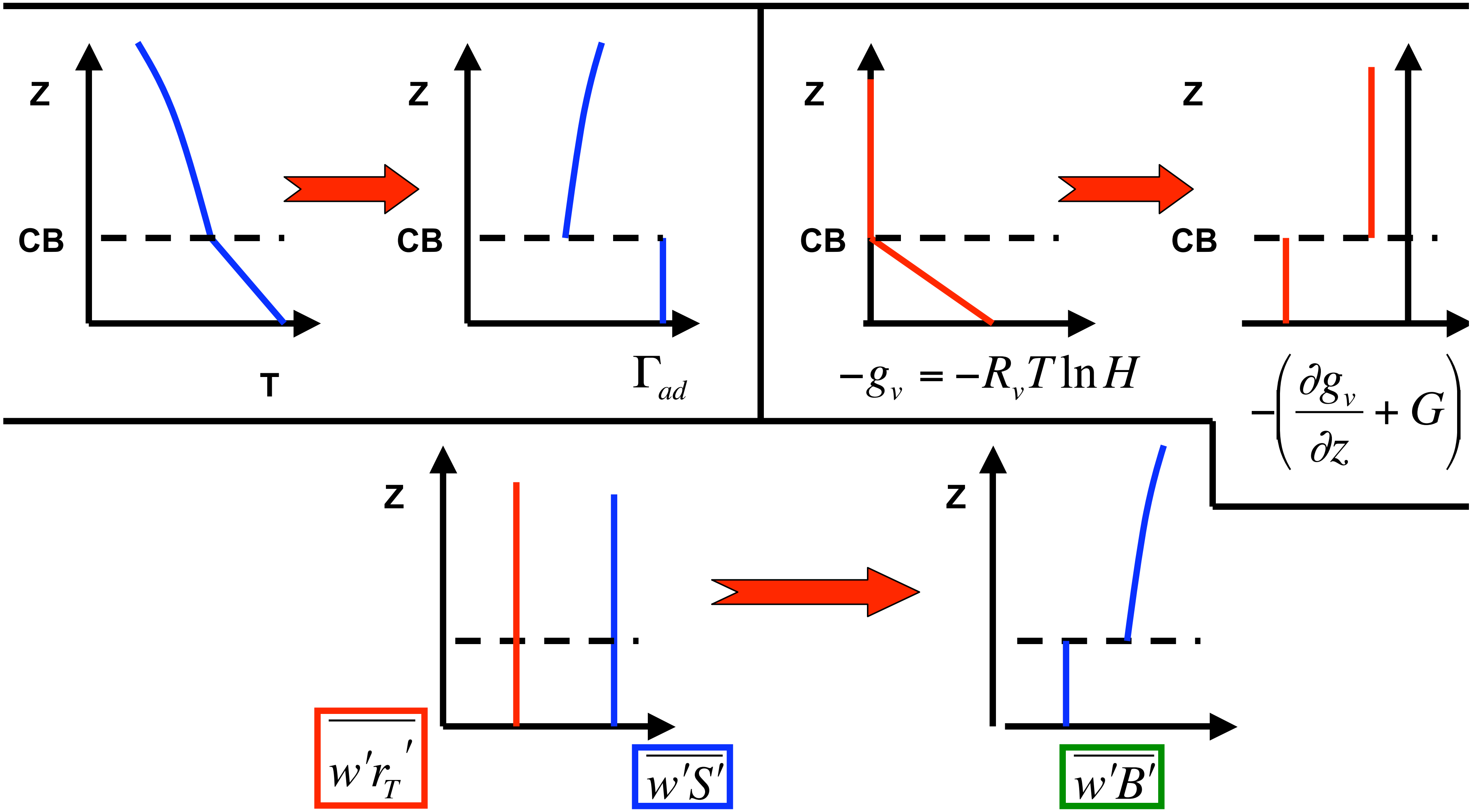
$$\boxed{\rho_0 \overline{w'B'} \cong \left(\frac{\partial B}{\partial S}\right)_{r_T,p} \rho_0 \overline{w'S'} + \left(\frac{\partial B}{\partial r_T}\right)_{S,p} \rho_0 \overline{w'r_T{}'}}$$

- The partial derivatives can be rewritten using the Maxwell relationships:

$$\left(\frac{\partial B}{\partial S}\right)_{r_T,p} = \rho_0 G \left(\frac{\partial \alpha}{\partial S}\right)_{r_T,p} = \rho_0 G \left(\frac{\partial T}{\partial p}\right)_{r_T,S} = -\left(\frac{\partial T}{\partial z}\right)_{r_T,S} = \Gamma_{ad}$$

$$\left(\frac{\partial B}{\partial r_T}\right)_{S,p} = \rho_0 G \left(\frac{\partial \alpha_d}{\partial r_T}\right)_{S,p} - G = \rho_0 G \left(\frac{\partial g_v}{\partial p}\right)_{r_T,S} - G = -\left(\frac{\partial g_v}{\partial z}\right)_{r_T,S} - G$$

$$\boxed{\rho_0 \overline{w'B'} \cong \Gamma_{ad}\rho_0 \overline{w'S'} - \left(\frac{\partial g_v}{\partial z} + G\right)\rho_0 \overline{w'r_T'}}$$



- After integration:

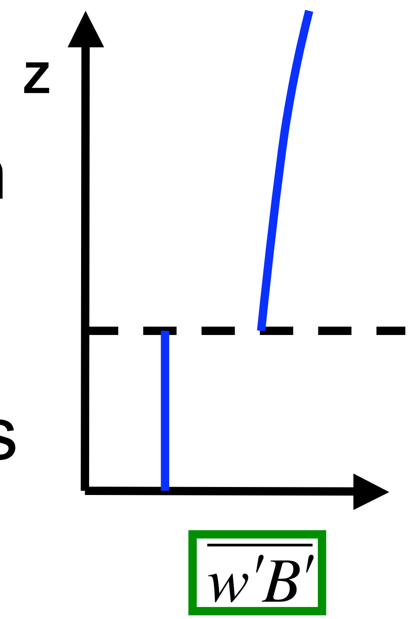
$$\frac{dKE}{dt} = \int_0^{\Delta z} \rho_0 \overline{w'B'} dz$$

$$= \underbrace{(T_{in} - T_{out})\rho_0 \overline{w'S'}}_{\text{Work done by a Carnot cycle}} + \underbrace{\left(g_{v,in} - g_{v,out}\right)\rho_0 \overline{w'r_T{}'}}_{\text{“water vapor penalty”}} - \underbrace{G\Delta z \rho_0 \overline{w'r_T{}'}}_{\text{geopotential energy gained by water}}$$

Work done by a mixed cycle

III. Why relative humidity affects work?

- IIIa: Enhanced buoyancy flux.
- Condensation at cloud base and re-evaporation at cloud top results in an enhanced buoyancy flux within the cloud layer.
- The depth of the cloud layer depends on the location of the cloud base, which is itself determined by the relative humidity.



IIIb: free energy!

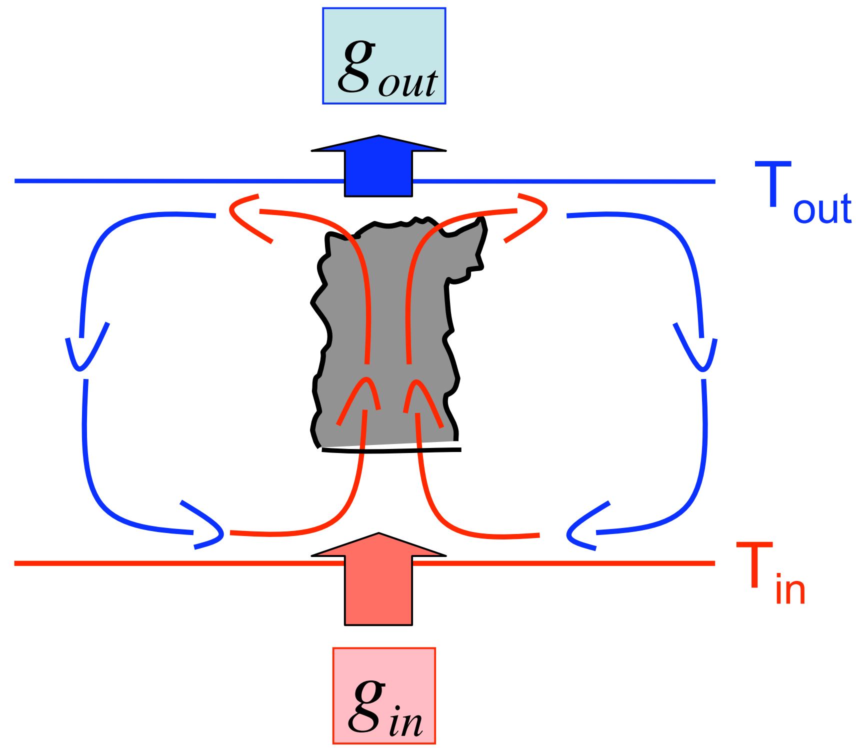
- "water vapor transport penalty" given by

$$\left(g_{v,in} - g_{v,out}\right)\rho_0 \overline{w' r_T{}'}$$

- Free energy depends primarily on relative humidity:

$$g_{in} = R_v T_{in} \ln H_{in}$$
$$g_{out} = R_v T_{out} \ln H_{out}$$

- Free energy of the water vapor decreases with height:



$$\boxed{g_{in} \leq g_{out}}$$

There is no free lunch…

- Its rate of change is given by

$$dg = sdT + \alpha dp$$

- For a reversible isothermal process, we have:

$$\boxed{\begin{aligned} dg - \alpha dp &= 0 \\ \Rightarrow \Delta g + W &= 0 \end{aligned}}$$

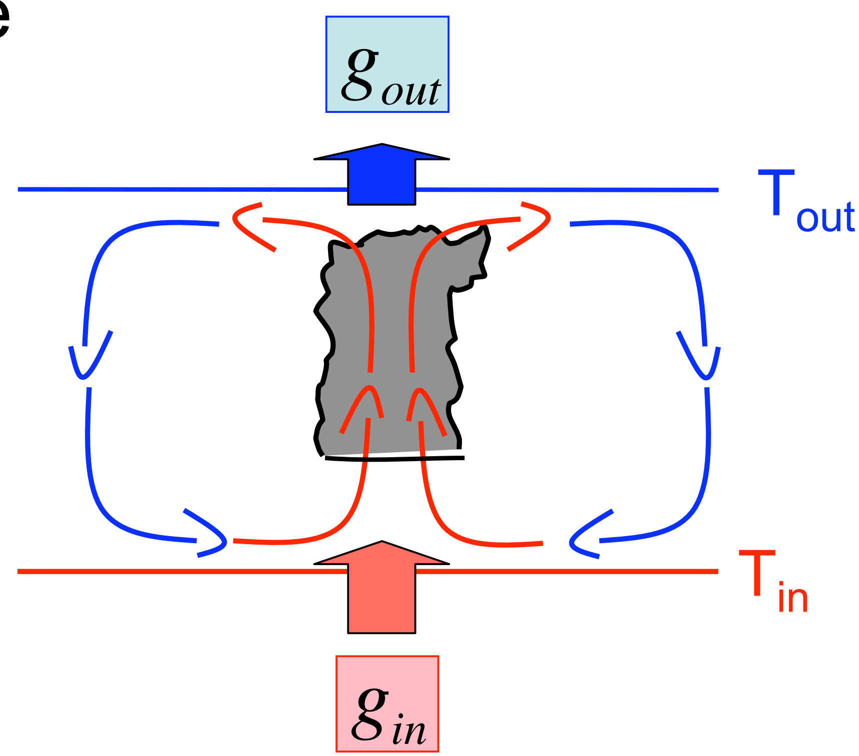
- The amount of work that can be extracted is equal to the reduction in free energy!
- And it is only possible to increase the free energy if work is exerted on the system

“water vapor transport penalty” due to an increase in the free energy as water is transported upward

$$\left(g_{v,in} - g_{v,out}\right)\rho_0 \overline{w'r_T'}$$

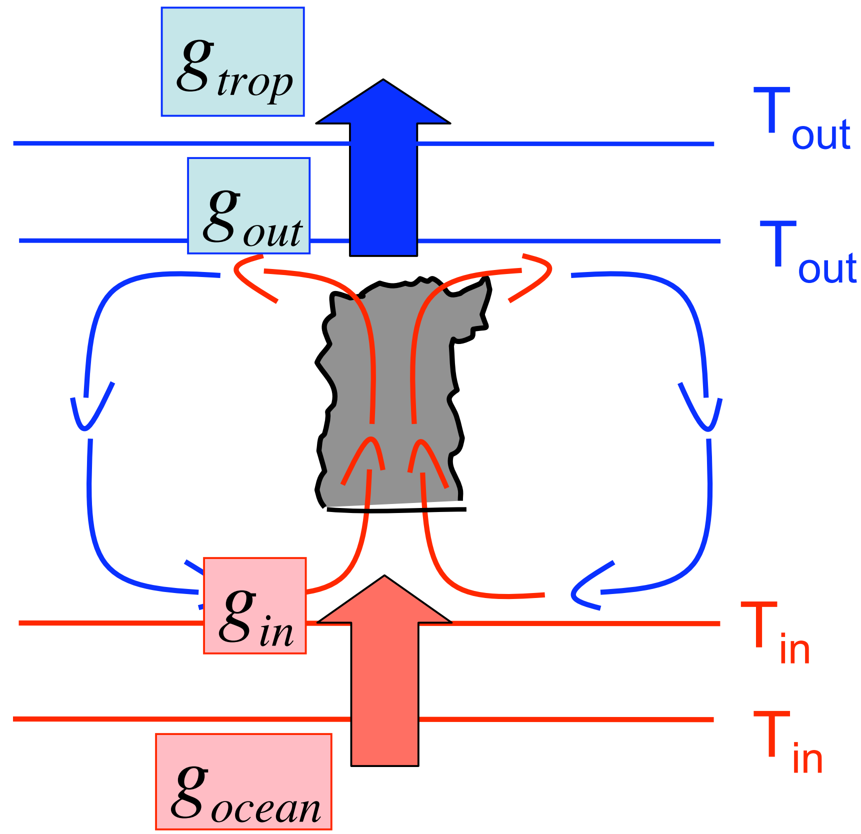
$$g_v = R_v T \ln H$$

- Free energy increase with height in an unsaturated ascent
- Bur is constant whenever the air is saturated



- IIIc. it is all tied to internal entropy production…

- In theory, the increase in free energy could be used later to generate mechanical work.
- In practice, it is wasted when water diffuses at the upper and lower boundary



Any additional work can be performed because there is a reduction in internal entropy production

IIc. Buoyancy flux in cumulus convection

$$\rho_0 \overline{w'B'} = \overline{\Gamma}_m \rho_0 \overline{w'S'} - g\rho_0 \overline{w'r_T{}'} - \left(\overline{\Gamma}_m \frac{L_v}{\overline{T}} - g\frac{R_v}{R_d} \right) \rho_0 \overline{w'(r - r^*)}$$

Carnot efficiency

Work required for lifting the water

"water vapor penalty"

Conclusion

- Convection acts as a heat engine that produces kinetic energy by transporting energy from warm to cold.
- However, it is not a Carnot cycle.
- The effects of moisture on the production of mechanical work are captured in an idealized steam cycle.
- In particular, expression for the buoyancy flux matches the work done by an idealized mixed steam-Carnot cycle.
- Key implication: everything else being equal, relative humidity increases the work done by convection.

- Latent and sensible heat flux:

$$Q_{lat} = L\Delta r_T$$

$$Q_{sen} = T_{in}\Delta S + (g_{in} - L)\Delta r_T$$

- Bowen ratio:

$$B = \frac{Q_{sen}}{Q_{lat}}$$

$$\boxed{\begin{aligned}\text{Efficieny } \eta &= \frac{B}{1+B}\eta_c + \frac{1}{1+B}\eta_H \\ &= \frac{T_{in} - T_{out}}{T_{in}} + \frac{1}{1+B}\frac{R_v T_{out}}{L}\ln\frac{H_{in}}{H_{out}}\end{aligned}}$$

The efficiency of a mixed cycle depends on both relative humidity and Bowen ratio.