The Wavelength Dependence of the Gross Moist Stability and the Scale Selection in the Instability of Column-Integrated Moist Static Energy

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ABSTRACT

Gross moist stability (GMS), a measure of how efficiently divergent flow exports column-integrated moist static energy (MSE), is a widely used quantity in current simplified models of the tropical mean circulation and intraseasonal variabilities such as the Madden–Julian oscillation (MJO), where it is often assumed to be constant. In this paper, it is shown, with cloud-system-resolving model experiments that incorporate feedback from the large-scale flow, that the GMS is smaller at longer wavelengths. The reason for this wavelength dependence is that temperature anomalies required to maintain a given divergent flow increase with wavelength. At long wavelengths, the required temperature anomalies become sufficiently strong to affect the shape of convective heating. As a consequence, the divergent flow is forced to be less top heavy in order to maintain the balance of momentum, heat, and moisture, as well as consistency with the behavior of cumulus convection. A simple model is constructed to illustrate this behavior. Given the ongoing theoretical efforts that view the MJO as resulting from instability in column-integrated MSE, the results presented here provide a planetary-scale selection for such instability, which is absent in current theoretical models that assume a constant GMS.

1. Introduction

Attention has recently been drawn to the scenario in which feedbacks from sources of column-integrated moist static energy (hereafter column MSE) render a moist convecting atmosphere unstable to perturbations in the column MSE. For example, in experiments with a cloud-system-resolving model (CSRM), Bretherton et al. (2005) found that a horizontally homogeneous, convecting atmosphere spontaneously selfaggregated into a moist region with enhanced column MSE and enhanced convection, surrounded by a dry region with reduced column MSE and suppressed convection. Their analyses of the column MSE budget showed that column MSE anomalies grow through their perturbations on the sources and sinks of column MSE. A column with a positive column MSE anomaly, for example, has enhanced convection, which drives a divergent flow that exports column MSE and damps the

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original anomaly. However, the enhanced convection also produces enhanced anvil cirrus cover as well as enhanced surface gustiness, which reduces the radiative cooling and increases the surface fluxes, respectively. An instability results when these additional sources of MSE exceed the increase in column MSE export. This instability has been considered as a candidate mechanism for tropical intraseasonal variabilities, such as the Madden–Julian oscillation (MJO), by a number of authors in both theoretical/conceptual models and diagnostics of comprehensive numerical models (Neelin and Yu 1994; Sobel et al. 2001; Fuchs and Raymond 2002; Bretherton et al. 2005; Fuchs and Raymond 2005, 2007; Raymond and Fuchs 2007; Maloney 2009; Raymond and Fuchs 2009; Sugiyama 2009a,b; Boos and Kuang 2010).

While the column MSE budget is a natural and convenient framework for diagnosing and expressing the instability in column MSE, it contains no information on cumulus dynamics beyond the conservation of energy. If changes in the column MSE sources, such as surface heat fluxes and radiative heating, are known or can be modeled/parameterized, the column MSE instability is largely determined by how efficient the divergent flow exports column MSE. A measure of this efficiency, named the gross moist stability (GMS), was first defined

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by Neelin and Held (1987) and has been widely used in conceptual models of tropical steady or quasi-steady motions [e.g., see the recent review of Raymond et al. (2009)].

In Neelin and Held (1987) and the studies referenced above, the GMS is assumed to be constant, corresponding approximately to the assumption of a fixed vertical shape for the divergent flow. While Neelin and Held considered the GMS as providing "a convenient way of summarizing our ignorance of the details of the convective and large-scale transients," Neelin (1997), followed by others (e.g., Neelin and Zeng 2000), advanced arguments to justify the use of a fixed vertical shape of the divergent flow. Since tropospheric temperature in convective regions is constrained to follow moist adiabats, Neelin argued that temperature differences from one region to another are characterized by differences between moist adiabats and, thus, have approximately a fixed vertical shape. This fixed shape of temperature anomalies, through hydrostatic balance, implies a fixed shape for the pressure perturbations, which, from horizontal momentum balance (with simple forms of momentum damping), implies a fixed vertical shape for the divergent flow. Barring substantial changes in the vertical profiles of MSE, this effectively fixes the GMS.

Enormous simplifications can thus be derived from the assumption that temperature departures from moist adiabats have negligible roles in driving the large-scale circulation. For steady flows or low frequency flows that may be considered as quasi steady, if the diabatic sources of the column MSE are known, evolutions of column MSE can be determined without specific knowledge of cumulus dynamics beyond its tendency to maintain moist adiabatic temperature structures. Moreover, the magnitude and the vertical distribution of convective heating are also known because in steady and quasisteady flows, it must balance the adiabatic cooling associated with the divergent flow. The effect of temperature anomalies on convection thus becomes redundant. Under the above assumptions, using the column MSE budget with a fixed GMS and neglecting horizontal temperature gradients as in the weak temperature gradient (WTG) approximation provide a simple and elegant framework for the tropical steady and quasi-steady circulation. This framework has been systematically studied by a number of authors (Sobel and Bretherton 2000; Bretherton and Sobel 2002, 2003; Peters and Bretherton 2005; Sugiyama 2009a,b).

Temperature profiles in the convective regions on average do indeed follow moist adiabats to within a kelvin or two and may not be distinguishable from moist adiabats given the instrumental errors (Xu and Emanuel 1989), not withstanding complications associated with the inclusion of latent heat of fusion and the retention of condensates (Williams and Renno 1993). However, in the tropics, horizontal temperature gradients above the boundary layer are small due to the effectiveness of gravity wave adjustment and horizontal temperature gradients in driving divergent flows. Therefore, when one considers horizontal temperature variations, the observed departure from the moist adiabat is not negligible compared to variations that follow the moist adiabat. Small temperature anomalies on the order of a kelvin can also have significant effects on convection because of the generally small buoyancy of convective updrafts (e.g., Kuang and Bretherton 2006; Kuang 2010; Tulich and Mapes 2010; among others). It is therefore questionable to neglect temperature departures from moist adiabats and to assume a fixed vertical shape for the divergent flow. Recent observational studies have indeed cast doubts on the generality of a fixed vertical shape for the vertical velocity profile, even for steady-state flows (Back and Bretherton 2006; Peters et al. 2008).

We shall therefore relax the assumption that temperature departures from moist adiabats do not significantly affect the large-scale divergence circulation. As will be seen, relaxing this assumption also necessitates considerations of how small temperature and moisture anomalies affect convection in order to obtain a closed problem. In this way, additional cumulus dynamics (beyond its tendency to maintain moist adiabats) enters the problem, resulting in a framework that is more elaborate than that based on the constant GMS assumption. However, we will show that this additional complexity provides a simple mechanism for the scale selection of the instability in the column MSE, which has eluded current theories that assume constant GMS (e.g., Bretherton et al. 2005; Raymond and Fuchs 2009;¹ Sugiyama 2009a).

This paper is structured as follows. Following a brief description of the model used, the experimental setup, and the method to represent coupling with the largescale flow (section 2) we examine how the GMS varies with wavelength using idealized experiments with a CSRM (section 3). In section 4, we present a simple model to illustrate the physical processes that give rise to this wavelength dependence. Implications of the scale selection of column MSE instability are described in section 5, while sections 6 and 7 present some discussion and the conclusions, respectively. An appendix is included to

¹ We are referring to the analytical model portion of Raymond and Fuchs (2009), described in their section 2.

describe how parameters used in the simple model were chosen.

2. Model, setup, and representation of feedbacks from large-scale flow

We use the System for Atmospheric Modeling (SAM) version 6.4, which solves the anelastic equations of motion (Khairoutdinov and Randall 2003). The prognostic thermodynamic variables are the liquid water static energy, total nonprecipitating water, and total precipitating water. We use a bulk microphysics scheme and a simple Smagorinsky-type scheme for the effect of subgrid-scale turbulence. The surface fluxes are computed using bulk aerodynamic formulae with constant exchange coefficients and a constant surface wind speed of 5 m s⁻¹ to eliminate any wind-induced surface heat exchange effect. The surface temperature is set to 29.5°C. For the experiments in this paper, the domain size is $384 \text{ km} \times 384 \text{ km}$ in the horizontal with a 2-km horizontal resolution. There are 64 stretched vertical grid points that extend from the surface to 32 km.

To allow for feedbacks from the large-scale circulation, the approach of Kuang (2008a, hereafter K08a) is used, which allows a limited-domain CSRM to be used to study the coupling between convection and twodimensional large-scale gravity waves of a single horizontal wavenumber. The derivation for an anelastic system was given in K08a. Briefly, we start with the 2D anelastic linearized perturbation equations of momentum, continuity, and hydrostatic balance (all equations are for the large-scale wave):

$$\overline{\rho}u_t' = -p_x' - \varepsilon \overline{\rho}u', \qquad (1)$$

$$\left(\overline{\rho}u'\right)_{x} + \left(\overline{\rho}w'\right)_{z} = 0, \tag{2}$$

and

$$p'_{z} = \overline{\rho}g\frac{T'}{\overline{T}},\tag{3}$$

where ε is the mechanical damping coefficient and all other symbols assume their usual meteorological meaning; the background mean variables are denoted with an overbar and the perturbation variables are denoted with a prime. By considering a single horizontal wavenumber *k* at a time, one can derive (see K08a for details)

$$\left\{ \left(\frac{\partial}{\partial t} + \varepsilon\right) \left[\overline{\rho}w'(x_0, z, t)\right]_z \right\}_z = -k^2 \frac{\overline{\rho}g}{\overline{T}} T'(x_0, z, t), \quad (4)$$

where x_0 marks the horizontal location of the vertical column modeled by the CSRM. Since this study is

concerned with disturbances with periods long compared to those of convectively coupled waves, we will neglect the time tendency and use the steady-state version of Eq. (4):

$$\left\{\varepsilon[\overline{\rho}w'(x_0, z, t)]_z\right\}_z = -k^2 \frac{\overline{\rho}g}{\overline{T}} T'(x_0, z, t), \qquad (5)$$

which filters out convectively coupled gravity waves. We compute T' as the deviation of the CSRM horizontally averaged (virtual) temperature profile from a reference (virtual) temperature profile and then use Eq. (5) to compute the large-scale vertical velocity. Effects of this vertical velocity are then included in the CSRM integration as source terms of temperature and moisture, applied uniformly in the horizontal as in Eqs. (8) and (9)of K08a. Integration of the CSRM produces new domainaveraged profiles of temperature and moisture, completing the coupling between convection and the large-scale flow. This approach was also used in Blossey et al. (2009) to examine the response of boundary layer clouds in the presence of feedbacks from the large-scale flow, except using the pressure coordinate and with the Coriolis force included. There are also other approaches for representing feedbacks from the large-scale flow in single-column models and limited-domain CSRMs (e.g., Sobel and Bretherton 2000; Shaevitz and Sobel 2004; Raymond and Zeng 2005). We chose to use the approach of K08a because it is based explicitly on the gravity wave equations, does not assume a given shape for the divergent flow, and does not require a special treatment in the boundary layer.

3. Results from CSRM experiments

The CSRM is first run in radiative-convective equilibrium (RCE) with prescribed time-invariant radiative cooling and no coupling with the large-scale dynamics for 150 days. The radiative cooling profile is the statistical equilibrium profile from an earlier long RCE integration of the model with interactive radiation computed using the National Center for Atmospheric Research Community Atmospheric Model radiation package. The last 50 days of this 150-day period were averaged to produce the reference profiles of temperature and moisture and the reference surface heat fluxes. Starting from day 150, we enable coupling with largescale dynamics using Eq. (5) and give surface heat fluxes a horizontally uniform offset at each time step so that the domain-averaged surface sensible and latent heat fluxes equal the reference values. This way, the domainaveraged surface heat fluxes are constant in time and precisely known. The model is then run for another 200 days. The steady-state quantities are computed by



FIG. 1. Precipitation time series, smoothed by a 25-day running mean, from experiments with horizontal wavelengths of (a) 4.0×10^4 , (b) 2.5×10^4 , and (c) 1.25×10^4 km. A 2.5-day Rayleigh damping is used. The three curves in each panel correspond to the cases with a perturbation surface heat flux of 2, 0, and -2 W m⁻².

averaging over the last 150 days. This will be referred to as the control run. Additional experiments are carried out, which are identical to the control except that after day 150 surface heat fluxes are offset at each time step so that the domain-averaged surface sensible and latent heat fluxes equal the reference values plus a perturbation. The Bowen ratio of the perturbation is the same as that of the reference surface heat fluxes. We performed experiments for horizontal wavelengths of 4.0×10^4 , 3.33×10^4 , 2.86×10^4 , 2.5×10^4 , 2.0×10^4 , 1.67×10^4 and 1.25×10^4 km with a vertically uniform momentum damping time ($1/\varepsilon$) of 2.5 days. As clear from Eq. (5), the coupling depends on k^2/ε , not k or ε individually. When a different ε is used, the results will be identical when k is adjusted to give the same k^2/ε .

In the present case, there is no large-scale horizontal advection so that

$$\frac{\partial \langle h \rangle}{\partial t} = \left\langle -w \frac{\partial h}{\partial z} \right\rangle + F, \tag{6}$$

where h is the MSE, the angle brackets denote a massweighted vertical integral, and F is the column MSE forcing. In our numerical experiments, F will be from the perturbation surface heat fluxes, although it can also arise from forcings in radiation and horizontal advection. In steady state, the additional MSE forcing must be balanced by the column integrated vertical MSE advection tendency (or column-integrated MSE export through an integration by parts in the vertical), and the anomalous latent heating from precipitation plus the anomalous surface sensible heat flux must balance the columnintegrated vertical advective tendency of the dry static energy. Therefore, we can estimate the GMS, defined here as the ratio of the steady-state column integrated vertical advective tendencies of MSE and dry static energy, as

$$M = \frac{\left\langle w \frac{\partial h}{\partial z} \right\rangle}{\left\langle w \frac{\partial s}{\partial z} \right\rangle} = \frac{F}{LP' + F_{\rm SH}},\tag{7}$$

where s is the dry static energy, L is the latent heat of vaporization, and P' is the precipitation anomaly. The perturbation surface sensible heat flux F_{SH} is in practice negligible since the Bowen ratio in the present case is about 0.09.

Figure 1 shows the precipitation time series from the CSRM experiments for three horizontal wavelengths, each with three perturbations surface heat fluxes. A 25-day running mean has been applied to the time series. With a wavelength of 4.0×10^4 km, the control run drifted after coupling with the large-scale flow is enabled (Fig. 1a).



FIG. 2. Steady-state precipitation for a range of perturbation surface heat fluxes for a horizontal wavelength of 4.0×10^4 km and Rayleigh damping time of 2.5 days.

This indicates a very small or likely negative GMS for the mean state so that small imbalances between radiative cooling and surface heat fluxes that exist even in the control run can result in substantial changes in the mean state. In other words, the RCE solution is an unstable equilibrium once larger-scale dynamics are included. To further illustrate this behavior, additional experiments with different surface heat flux perturbations were carried out. Figure 2 shows the resulting steady-state precipitation as a function of the surface heat flux perturbation. It appears that the system bifurcates from the original state under small perturbations to new mean states that have more positive GMS (smaller slope in the P'-F curve). The same behavior is seen to a lesser extent for a wavelength of 3.3×10^4 km. The system thus appears to have two equilibria at these wavelengths. The two equilibria found here are different and much less dramatic than the multiple equilibria noted by Sobel et al. (2007) in a single-column model and by Sessions et al. (2010) in a limited-domain CSRM, where there is an equilibrium that has no precipitation.

At shorter wavelengths $(3 \times 10^4 \text{ km} \text{ onward})$, the control runs do not drift when coupling with the largescale flow is enabled (Figs. 1b,c) and responses to surface heat flux perturbations are approximately linear for the size of perturbations that we apply $(1-2 \text{ W m}^{-2})$ (Fig. 3). At the shortest wavelengths considered here, such as the 1.25×10^4 -km case shown in Fig. 1c, there are significant oscillations in the precipitation. Composite structures of these oscillations (not shown) are similar to those of convectively coupled waves reported in K08a except for a $\pi/2$ shift in the phasing between temperature anomalies and vertical velocity (and convective heating



FIG. 3. As in Fig. 2, but for a horizontal wavelength of 2.5×10^4 km.

and moisture) anomalies. That convectively coupled disturbances can develop even when the time tendency term in Eq. (4) is neglected is interesting. However, this is not the focus of this study and will not be pursued further in this paper.

Steady-state precipitation responses per 1 W m⁻² surface heat flux perturbation for the different wavelengths are summarized in Fig. 4a. Figure 4b shows the GMS estimated from Eq. (7) (circles). Steady-state precipitation responses to surface heat flux perturbations are stronger and the GMS is lower at longer wavelengths. Cases of 4.0×10^4 km and 3.33×10^4 km are not shown here. While it is clear that their GMS are either small or negative, their GMS cannot be reliably estimated for the original mean state because of the drift in the control run.

Vertical profiles of the steady-state mean pressure velocity (Fig. 5a) show that the large-scale vertical velocity is more top heavy at shorter wavelengths. This is particularly clear when the pressure velocity profiles are normalized to have a peak amplitude of 1 (Fig. 5b). It is well known that, since the vertical MSE gradient is negative in the lower troposphere and positive in the upper troposphere, a more top-heavy vertical velocity profile is more efficient in reducing column MSE, leading to a greater GMS. While changes in the GMS can be due to changes in the mean and in the transients, the tendency for GMS to increase at shorter wavelengths is well captured by changes in the time-mean circulation, as seen in Fig. 4b, which also shows the GMS associated with the mean circulation (asterisks),

$$\left\langle \overline{w} \frac{\partial \overline{h}}{\partial z} \right\rangle / \left\langle \overline{w} \frac{\partial \overline{s}}{\partial z} \right\rangle,$$



FIG. 4. (a) Steady-state precipitation anomalies associated with a 1 W m⁻² forcing in surface heat fluxes and (b) the gross moist stability (GMS) (circles) as functions of planetary wavenumber. Asterisks in (b) are the GMS computed based on the steady-state vertical profiles.

where the overbar denotes time average. While the timeaveraged h and s profiles in the perturbation runs differ somewhat from those of the control run, the results shown in Fig. 4b are unchanged when the time-averaged h and s profiles from the control case are used instead. Therefore, the tendency for GMS to increase at shorter wavelengths is caused by changes in the vertical velocity profiles rather than changes in the time-averaged h and sprofiles.

The question now is why vertical velocity profiles are less top heavy at longer wavelengths. We argue that it is because of the stronger temperature anomalies required to maintain a given divergent flow at longer wavelengths, as seen in Eq. (5) and Fig. 5c. Take a top-heavy vertical velocity profile as an example. From Eq. (5), we see that a temperature anomaly that is warmer in the upper troposphere than in the lower troposphere is required to maintain this vertical velocity profile. At short wavelengths, the required temperature anomalies are small and their effect on convection may be safely neglected (as in the WTG approximation). Let us suppose that a certain top-heavy vertical velocity profile is



FIG. 5. (a) Steady-state pressure velocity, (b) normalized pressure velocity, and (c) temperature anomalies, associated with a 1 W m⁻² forcing in surface heat fluxes for various wavelengths.

obtained in this limit. Now, as we move toward longer horizontal wavelengths, stronger temperature anomalies are required to maintain this vertical velocity profile. At sufficiently long wavelengths, the temperature anomalies required will be strong enough to significantly discourage convective updrafts from reaching the upper troposphere and making convective heating more bottom heavy. This will reduce the top-heaviness of the vertical velocity profile since, in steady state, adiabatic cooling associated with the vertical velocity profile must be consistent with the convective heating. In the next section, we present a simple model to illustrate the above ideas more clearly.

4. A simple model

Like many previous models (e.g., Mapes 2000; Khouider and Majda 2006; Kuang 2008b, hereafter K08b), the present simple model has two vertical modes for the free troposphere and a subcloud layer. We approximate temperature anomalies in the free troposphere with two vertical structures. One is constant with height and the other linear with height:

$$\frac{\overline{\overline{\rho}}}{\overline{T}}T' = \frac{\overline{\overline{\rho}}_0}{\overline{T}_0} \sum_{j=1}^2 T_j \Phi_j(z), \tag{8}$$

where $\Phi_1(z^*) = 1$ and $\Phi_2(z^*) = 1 - 2z^*$ are the basis functions, $z^* = z/z_T$ is the nondimensional height, z is the height, and $z_T = 13.5$ km is the depth of the strongly convecting troposphere; $\overline{\rho}(z)$ and $\overline{T}(z)$ are the reference density and temperature profiles, respectively. We have defined density and temperature scales $\overline{\rho}_0 = 1$ kg m⁻³ and $\overline{T}_0 = 300$ K to make the basis functions Φ_j dimensionless. These two vertical modes are chosen empirically to approximate the temperature anomalies seen in Fig. 5c, which facilitates parameter estimates described in the appendix. The basic behaviors are unchanged if one chooses to use the first two vertical modes of the gravity wave equation (assuming rigid-lid boundary conditions at the top and the bottom of the troposphere).

From Eq. (5) and the heat equation,

$$\frac{\partial T'}{\partial t} + w' \left(\frac{d\overline{T}}{dz} + \frac{g}{c_p} \right) = J', \tag{9}$$

where J' is the anomalous convective heating. Assuming rigid-lid boundary conditions at the top and the bottom, we have

$$\overline{\rho}w'\left(\frac{d\overline{T}}{dz} + \frac{g}{c_p}\right)_0 = \overline{\rho}_0 \sum_{j=1}^2 w_j \Omega_j(z)$$

and

$$\overline{\rho}J'\left(\frac{d\overline{T}}{dz} + \frac{g}{c_p}\right)_0 = \overline{\rho}_0 \sum_{j=1}^2 J_j \Omega_j(z) \left(\frac{d\overline{T}}{dz} + \frac{g}{c_p}\right), \quad (10)$$

where $\Omega_1(z) = 4z^*(1 - z^*)$ and

$$\Omega_2(z) = 21 \left(z^{*3} - \frac{3}{2} z^{*2} + \frac{1}{2} z^* \right)$$

are the basis functions of adiabatic cooling and convective heating and are scaled to have a peak amplitude of one.² We have defined a stratification scale

$$\left(\frac{d\overline{T}}{dz} + \frac{g}{c_p}\right)_0 = 3.5 \times 10^{-3} \text{ K m}^{-1}$$

to make the basis functions Ω_j dimensionless. The variables w_j have the dimension of heating rates (with units of, e.g., K day⁻¹).

Projecting Eq. (5) onto the two modes gives

$$w_j = \frac{k^2}{\varepsilon} c_j^2 T_j, \tag{11}$$

where
$$c_1 = \left[\frac{H_T^2 N_0^2}{8}\right]^{1/2} \approx 51 \text{ m s}^{-1}$$
 and
 $c_2 = \left[\frac{H_T^2 N_0^2}{63}\right]^{1/2} \approx 18 \text{ m s}^{-1}.$

We have defined $N_0^2 = \overline{T}_0^{-1} g(d\overline{T}/dz + g/c_p)_0$. Equation (11) gives the temperature anomalies that are required to sustain a given divergent flow (represented by the vertical velocity profile) against momentum damping, and we see that temperature anomalies required to sustain a given divergent flow scale as wavelength squared and are greater for the second mode than for the first mode.

The modal version of the heat balance [Eq. (9)], assuming steady state, is

$$w_i = J_i. \tag{12}$$

This simply states the balance between adiabatic cooling (due to the vertical motion) and convective heating. In

 $^{^2}$ With a factor of 21 this is not exactly, but very closely, true for $\Omega_2.$

Symbol	Normative values	Unit	Description
$m_1/m_q m_2/m_q$	1, 0.5	day	Combined (advective and convective) free troposphere q tendencies per unit L and L normalized by that per unit a^+ [Eq. (13)]
γ_T	1	day^{-1}	Dependence of second-mode heating on T_2 [Eq. (14)]
γ_q	0.7	day^{-1}	Dependence of second-mode heating on saturation deficit [Eq. (14)]
c_1, c_2	1, ¹ /3	50 m s^{-1}	Efficiencies of the first- and second-mode temperature anomalies in driving the divergent flow [Eq. (11)]
ε	1/2.5	day^{-1}	Momentum damping coefficient [Eq. (11)]

TABLE 1. A summary of parameters used in the simple model.

steady state, we also have balanced free troposphere moisture budget:

$$0 = \frac{dq}{dt} = m_1 w_1 + m_2 w_2 - m_q q^+.$$
(13)

We have defined $q^+ \equiv q - q^*$, where q and q^* are the free tropospheric specific humidity and saturation specific humidity, respectively. A negative q^+ indicates a stronger saturation deficit in the free troposphere. The parameters m_1 and m_2 represent the combined effects of vertical moisture advection and convective drying associated with w_1 and w_2 (note that in steady state $w_{1,2}$ = $J_{1,2}$). The q^+ term represents the fact that, when the free troposphere is dry, there is more evaporation of rain and more detrainment of cloud water, both contributing to moistening of the free troposphere. In K08b the emphasis was on convectively coupled waves, where free tropospheric moisture is the main control on the secondmode convective heating J_2 . In that case, there is a strong association between q^+ and J_2 and the q^+ term is absorbed into the J_2 term. In the present study, as we will emphasize later, temperature effects on the secondmode convective heating become important at long wavelengths. This necessitates separating the effects of q^+ and J_2 (or w_2).

The above budget (momentum, free troposphere heat, moisture) equations are augmented by an equation for the second-mode convective heating:

$$J_2 = -\gamma_q q^+ - \gamma_T T_2. \tag{14}$$

This equation states that there is more heating (cooling) in the upper (lower) troposphere when the free tropospheric saturation deficit is low (i.e., the free troposphere is moister) and when the temperature anomaly in the lower troposphere is more positive than that in the upper troposphere (and vice versa). Effects of temperature stratification and/or free troposphere moisture have been included in a number of previous simplified models with two vertical modes in the context of convectively coupled waves (e.g., Mapes 2000; Khouider and Majda 2006; K08b). Raymond and Sessions (2007) also argued for the effect of temperature stratification anomalies on the vertical profiles of convective heating and the divergent flow in the problem of tropical cyclogenesis. In a similar spirit, Back and Bretherton (2009) also argued, using sea surface temperature as a proxy for boundary layer temperature and employing the WTG approximation in the free troposphere, that relatively small differences in the conditional instability between the central–eastern Pacific and the western Pacific warm pool could be important in explaining their different vertical motion profiles.

Combining Eqs. (11), (12), (13), and (14), we can determine the shape of the vertical velocity profile. More specifically, $A \equiv -w_2/w_1$, a measure of the top heaviness of the vertical velocity profile, is given by

$$A \equiv -\frac{w_2}{w_1} = \gamma_q \frac{m_1}{m_q} / \left(1 + \frac{\varepsilon \gamma_T}{k^2 c_2^2} + \gamma_q \frac{m_2}{m_q} \right).$$
(15)

Note that in previous work (e.g., Neelin 1997), the shape of the vertical velocity profile is fixed, whereas in Eq. (15) this shape is a function of the horizontal wavenumber. The top heaviness of the vertical velocity profile from Eq. (15), with parameter estimates made in the appendix and listed in Table 1, is shown in Fig. 6 together with those from the CSRM experiments. Note that the CSRM results are effectively a straight line and therefore have only 2 degrees of freedom. Given the crudeness in the parameter estimates, the excellent agreement between simple model and the CSRM results seen in Fig. 6 is fortuitous. Nevertheless, since the parameter estimates are made with no direct reference to the wavelength dependence, Fig. 6 is a meaningful demonstration that the simple model reproduces the general trend in the GMS seen in the CSRM experiments over the correct wavenumber range. The formulation in Eq. (15) does not permit bottom-heavy convective heating profiles (positive w_2/w_1). However, this result depends on details of the convective treatment. For example, if instead of Eq. (14) we use



FIG. 6. Top heaviness of the vertical velocity profile (defined as $-w_2/w_1$) as a function of wavenumber from the CSRM (circles) and the simple model (dashed).

$$J_2 = -\gamma_q q^+ - \gamma_T \frac{T_1 + T_2}{2},$$
 (16)

emphasizing the effect of lower tropospheric temperature anomalies based on results from Kuang (2010) and Tulich and Mapes (2010), then the convective heating profile can become bottom heavy at very long wavelengths. The general tendency for a less top-heavy vertical velocity at longer wavelengths, however, is robust to such changes.

Using the simple model, we now expand on the ideas discussed at the end of section 3 to explain the wavelength dependence of the top heaviness of the vertical velocity profile seen in Eq. (15). A schematic is shown in Fig. 7 to aid the discussion.

At sufficiently short wavelengths T anomalies will be small enough, Eq. (11), that their effects on convection can be neglected in Eq. (14). As an example, we shall consider the case in which w_1 (and hence the precipitation anomaly) is positive; the negative precipitation case can be considered similarly. In this case, the free troposphere will be moist (i.e., q^+ will be positive) and w_2 will be negative, which implies a top-heavy profile. This is because, if q^+ is negative, then w_2 is positive from Eq. (14) (i.e., convective heating is bottom heavy when the free troposphere is dry and temperature anomalies are small). In this case, both convection and large-scale vertical advection are moistening the free troposphere (i.e., $m_1w_1 + m_2w_2 - m_aq^+ > 0$), and the moisture balance cannot be satisfied. Next, consider the same vertical velocity profile (i.e., same values for w_1 and w_2) but for a long wavelength. The free troposphere moisture

balance, Eq. (13), is wavelength independent. This implies that the saturation deficit will remain the same. However, with a long wavelength the temperature anomalies required to maintain the divergent circulation become sizable. In particular, a sizable temperature anomaly that is positive (negative) in the upper (lower) troposphere is required to sustain a top-heavy divergent circulation. Such a temperature anomaly is not favorable for cloud updrafts to reach the upper troposphere and will lead to more bottom-heavy convective heating. The more bottom-heavy convective heating is no longer consistent with the originally assumed vertical velocity profile, and consistency can be restored only when the vertical velocity is also adjusted to a more bottom-heavy profile. In this way, the horizontal momentum balance plus the continuity and hydrostatic balance [Eq. (11)], the free tropospheric moisture balance [Eq. (13)], and the free tropospheric heat balance [Eq. (12)] together with the dependence of convection on temperature and moisture perturbations [Eq. (14)] produce the wavelength dependence of the top heaviness of the divergence circulation and, hence, the GMS.

We also have the column MSE balance

$$F - g_1 w_1 - g_2 w_2 = 0, (17)$$

where g_1 and g_2 are the GMS associated with the first and second vertical modes, respectively. The MSE balance in the subcloud layer can be deduced from Eqs. (12), (13), and (17) from MSE conservation. Combining Eqs. (15) and (17), we can solve for the response to a forcing in surface heat fluxes. For simplicity, we set $g_1 = 0$ based on discussions in the appendix. The precipitation response to a forcing *F* is then simply $F/(|g_2|A)$ in which *A* is the measure of the top heaviness of the vertical velocity profile defined in Eq. (15).

While not the emphasis here, other forms of forcing such as radiative heating or horizontal moisture advection can also be accommodated in the simple model by adding corresponding terms in the moisture and heat equations. For example, the response to an anomalous radiative heating that takes the shape of the first mode can be computed by including a radiative heating term R_1 in the heat balance equation for the first mode:

$$w_1 = J_1 + R_1. (18)$$

The column MSE budget is now

$$fR_1 - g_1w_1 - g_2w_2 = 0, (19)$$

where a factor f is used so that fR_1 is the column MSE forcing due to R_1 . When g_1 is set to zero, the expression

For short wavelengths (i.e. large k)



For long wavelengths (i.e. small k)



FIG. 7. Schematic of the wavelength dependence of top heaviness of the vertical velocity profile; see text for discussion.

for the top heaviness of the vertical velocity profile is identical to that due to surface heat flux anomalies, Eq. (15). We have confirmed this wavelength dependence in the response to first-mode radiative heating anomaly with CSRM simulations.

The response to a radiative heating anomaly that takes the shape of the second mode warrants additional discussion. Its effect can be included by adding a radiative heating term R_2 to the heat balance for the second mode:

$$w_2 = J_2 + R_2. (20)$$

Since there is no net column MSE source, the column MSE budget is now

$$-g_1 w_1 - g_2 w_2 = 0. (21)$$

Again setting $g_1 = 0$, we have the solution

$$\begin{split} w_1 &= J_1 = \frac{m_q R_2}{m_1 \gamma_q}; \quad w_2 = 0; \quad J_2 = -R_2; \quad q^+ = \frac{R_2}{\gamma_q}; \\ T_1 &= \frac{m_q R_2}{m_1 \gamma_q} \frac{\varepsilon}{k^2 c_1^2}; \quad T_2 = 0. \end{split} \tag{22}$$

The simple model thus shows that, when there is anomalous radiative heating in the lower troposphere and an equal amount of anomalous radiative cooling in the upper troposphere so that there is no net column MSE forcing, the resulting steady state has an increase in precipitation and a warmer and more humid (in terms of saturation deficit) free troposphere—a behavior that is also confirmed by CSRM experiments. This behavior is not directly captured in the MSE budget/GMS framework [e.g., Eq. (7)] since there is no net column MSE forcing associated with this radiative heating profile. However, one may rephrase the question to make use of the MSE budget/GMS reasoning: one could consider the combined effect of the anomalous radiative heating/ cooling couplet and its corresponding steady-state vertical advection of heat and moisture as the external forcing. The problem is then phrased in terms of the response to a column MSE forcing due to the vertical moisture advection associated with the circulation, the vertical heat advection of which cancels out the radiative heating/cooling couplet. Phrased in this way, MSE budget/GMS budget reasoning indicates that precipitation should increase when there is anomalous radiative heating in the lower troposphere and an equal amount of anomalous radiative cooling in the upper

troposphere, and vice versa. The above discussion, however, does illustrate that the simple model constructed here describes more generally the behavior of the system than the GMS viewpoint.

5. Scale selection in column MSE instability

While we have assumed a steady state in the previous section, the results hold for a quasi-steady state or slowly evolving cases where the time tendencies are small. A direct implication is that instability of column MSE should favor planetary scales. Consider a simple form of the column MSE instability, where the column MSE source varies linearly with precipitation anomalies:

$$F = \alpha L P', \tag{23}$$

as done in number of previous studies (e.g., Bretherton and Sobel 2002; Sobel and Gildor 2003; Sugiyama 2009b). From the column MSE budget, Eq. (6), and the definition of GMS, Eq. (7), assuming approximate balance between adiabatic cooling and convective heating anomalies and neglecting the sensible heat flux anomalies, we have

$$\frac{\partial \langle h' \rangle}{\partial t} = (\alpha - M)LP'.$$
(24)

Since positive precipitation anomalies P' are expected for positive column MSE anomalies (and vice versa), instability in column MSE will arise when $\alpha - M$ is positive [sometimes referred to as a case of negative effective gross moist stability, e.g., Raymond et al. (2009)]. For $\alpha \sim 0.25$ as used in, for example, Sobel and Gildor (2003) (which accounts for the effects of both radiative and surface heat flux feedbacks), our results indicate that $\alpha - M$ increases with wavelength and becomes positive for wavelengths longer than $\sim 2.0 \times 10^4$ km.

On the other hand, however, the feedback from the large-scale flow weakens at longer wavelengths. At long wavelengths it is reasonable to consider temperature anomalies to be proportional to column MSE anomalies. From Eq. (5) and the approximate balance between adiabatic cooling and anomalous precipitation, we have

$$\frac{dP'}{d\langle h'\rangle} \propto k^2. \tag{25}$$

A limiting case is for k = 0 in which there is no feedback from the large-scale flow and the precipitation anomaly will be zero regardless of anomalies in the column MSE (we have again neglected the role of surface sensible heat flux). Combining Eqs. (24) and (25), we have

$$\frac{\partial P'}{\partial t} \propto (\alpha - M)k^2 LP'.$$
(26)

Because $\alpha - M$ increases with wavelength and becomes positive at planetary or longer scales and because the feedback from the large-scale flow weakens at longer wavelengths, the growth rates of the column MSE instability peak at planetary scales.

Column MSE instability arising from radiative feedbacks has an additional dependence on the shape of the radiative heating anomalies. If the radiative heating anomaly is of the same sign and magnitude in the upper and lower troposphere, the same planetary-scale selection in column MSE instability should apply because precipitation response (and the GMS) to this type of radiative heating anomalies has a wavelength dependence similar to that for surface heat flux anomalies. However, if in a region of positive MSE anomaly and enhanced convection, radiative cooling is reduced more in the lower troposphere than in the upper troposphere, the atmosphere is more unstable to column MSE instability because this radiative cooling anomaly can feed back to the column MSE and precipitation anomalies more strongly, as discussed at the end of section 4. It is interesting to note that the same radiative feedback profile (where in a region of enhanced convection, radiative cooling is reduced more in the lower troposphere than in the upper troposphere) will reduce the (direct) stratiform instability for convectively coupled waves because it reduces the top-heaviness of the convective heating profile (e.g., Mapes 2000; K08b).

6. Discussion

In our discussion, we have deliberately avoided the term "moisture mode," which has often been used for modes arising from the column MSE instability (e.g., Sobel et al. 2001; Raymond and Fuchs 2009; Sugiyama 2009a,b). The term "moisture mode" has been used in part because perturbations in column MSE are largely from moisture anomalies, but it also conveys the sense of a subsidiary role for temperature anomalies, which have often been neglected by making the WTG approximation. However, we have shown that temperature anomalies, while a secondary contributor to column MSE anomalies, are important in setting the shape of convective heating and the divergent flow, and hence the GMS. Therefore, we prefer the term "column MSE instability" over the term "moisture mode."

In the simple model described in section 4, we assumed (quasi-) steady state to exclude convectively coupled waves and focus on the column MSE instability. Modes from the column MSE instability are fundamentally different from convectively coupled waves and do not propagate by (convectively modified) gravity wave dynamics. The planetary-scale column MSE instability modes and convectively coupled waves can be included in a single theoretical model by modifying the models of K08b or Andersen and Kuang (2008). This can be done by replacing Eq. (7) of K08b with the timevarying version of Eq. (13) here to separate the effects from the secondary-mode convective heating and from the saturation deficit anomalies, and by replacing Eq. (20) of K08b with Eq. (14) here to allow temperature anomalies to affect the shape of convection.

Finally, we would like to note that absolute values of GMS are expected to depend on the CSRM and configurations used and the mean state for which they are computed. However, the processes that lead to the wavelength dependence of the GMS described here should be robust.

7. Conclusions

Gross moist stability-a measure of how efficient divergent flow exports column MSE-has been often assumed as constant in theoretical models of the tropical circulation. Using CSRM experiments that incorporate feedback from the large-scale flow, we have shown that the GMS is smaller at longer wavelengths. The reason for this wavelength dependence is that temperature anomalies required to maintain a given divergent flow increase with wavelength. At sufficiently long wavelengths, the temperature anomalies become sufficiently significant that they affect the shape of convective heating. As a consequence, the divergent flow is forced to be less top heavy in order to maintain momentum, heat, and moisture balance as well as consistency with the behavior of cumulus convection. A simple model is constructed to illustrate this behavior. Given the ongoing theoretical efforts to explain the MJO as a column MSE instability (Neelin and Yu 1994; Sobel et al. 2001; Fuchs and Raymond 2002, 2005, 2007; Raymond and Fuchs 2007, 2009; Sugiyama 2009a,b), our results provide the planetary-scale selection that has been lacking in these models that assume a constant GMS.

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APPENDIX

Parameter Estimates of the Simple Model

The parameter estimates are made as follows. At each wavelength, we project the vertical velocity and temperature anomaly profiles onto the modes following Eqs. (8) and (10). This gives estimates of w_1, w_2, T_1 , and T_2 . We also calculate q^+ anomalies for each wavelength as the mass-weighted averages of the difference between specific humidity and saturation specific humidity between 1 and 10 km. In Fig. A1, we plot w_1 and w_2 as a function of q^+ . We see that w_2 is roughly constant with q^+ . Based on Eq. (13) and the slope of w_1 versus q^+ we chose $m_1/m_q = 1$ day. The intercept of the w_1-q^+ line with the y axis (i.e., when q^+ is zero) is roughly 0.01 K day⁻¹, while for w_2 it is roughly -0.02 K day⁻¹. Based on this, we chose $m_2/m_1 \sim 0.5$. In Fig. A2, we see that w_2 (and hence J_2) stays approximately constant while T_2 varies approximately linearly with q^+ . Given the relationship adopted in Eq. (14), the slope of the T_2 versus q^+ curve suggests $\gamma_{\rm T} \sim 1 \text{ day}^{-1}$. The intercept of the T_2 versus q^+ curve with the x axis is ~0.15 K. Since w_2 is about 0.02 K day⁻¹, this suggests that $\gamma_q \sim 1.3$ day⁻¹. Since all the cases shown are for the same column MSE forcing F and it appears that w_2 remains approximately constant in all the cases while w_1 varies substantially, from Eq. (17), it seems reasonable to choose $g_1 = 0$ for simplicity. The parameter estimates are listed in Table 1.

humidity surplus anomaly (q^+) from the CSRM simulations.





FIG. A2. As in Fig. A1, but of T_2 and w_2 .

As is clear from the above description, these are only educated guesses that approximately reproduce the relationships between various variables seen in the CSRM simulations. In the future, we hope to better quantify the parameters by first computing the linear response function matrix following the approach of Kuang (2010) and then project it onto the two-vertical-mode framework. At the moment, the linear response function matrix is not yet accurate enough for this purpose.

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