

Building a Fish: The Biology and Engineering Behind a Bioinspired Autonomous Underwater Vehicle

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Introduction

True autonomy in a robotic system comes from being able to interpret environmental stimuli, overcome destabilizing perturbations, and make motor control modifications depending on rapidly changing, unpredictable circumstances, without the aid of external inputs. In the fluid environment, both sensory information and destabilizing perturbations can come from any angle. Thus, autonomy in a fluid environment is a three-dimensional problem, requiring multiple sensors and control surfaces.

In the biological world, fishes resolve this three-dimensional problem using active modulation of their multiple, flexible fins that can vector forces in three dimensions to control body position. The presence of multiple fins (Figure 1) arrayed around the body to control fluid forces is a hall-

ABSTRACT

Bioinspired robotic designs have proven to be effective models for autonomous vehicles as well as important research tools in comparative biomechanics. Here we review the process by which we investigated the functional morphology and biomechanics of fish fins using live fish experiments and computational modeling; created and validated independent fins with regard to biological properties like stiffness, kinematics, and fluid dynamics; and constructed an autonomous underwater vehicle with a sensory feedback system to respond to perturbations.

Keywords: fish robot, fin design, bioinspired, underwater vehicle

mark of fish functional design. Fins evolved over hundreds of millions of years, likely as a result of selective pressure for fish to become more efficient, maneuverable swimmers; the evolution of bony fishes is coincident with the evolution of new habitats, such as corals, maneuvering around which is desirable for hiding from predators, finding prey, and having a safe place to rear young (Flammang, 2014). If one assumes that only competent morphologies succeed through natural selection, then evolution has acted as a sort of testing and refinement procedure that provides us with a strong starting point from which to learn and optimize for robotic design.

In order to engineer an autonomous underwater vehicle that has the locomotor agility of a fish, it is useful to have a comprehensive understanding of how fishes create and control forces with their fins. Following the extensive work that has been done understanding the kinematics and hydrodynamics of the myriad of swimming behaviors of the bluegill sunfish (*Lepomis macro-*

chirus; Chadwell et al., 2012; Dong et al., 2010; Flammang et al., 2013; Flammang & Lauder, 2008, 2009, 2013, 2016; Gibb et al., 1994; Higham et al., 2005; Jayne et al., 1996; Lauder et al., 2002; Peng et al., 2007; Standen & Lauder, 2005; Tytell, 2006; Tytell & Lauder, 2008), it seemed an appropriate candidate for a bioinspired design.

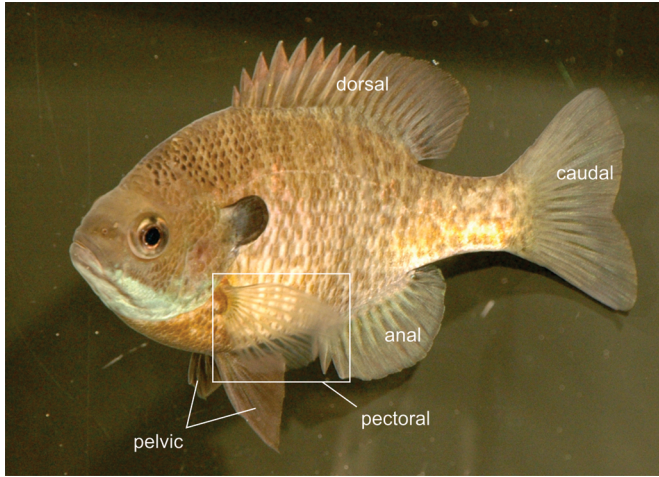
The goal of this research was twofold: to engineer a biorobotic fish that would serve as an experimental platform to investigate the mechanics, sensing, and control of multifin swimming and to use the testable platform to optimize an autonomous underwater vehicle that would be robust to perturbations in a natural setting. In this paper, we provide a review of this research program, starting from bluegill sunfish locomotor biomechanics and ending with a robotic system.

Functional Morphology of Fish Fins

The fins of teleost fishes are extremely flexible, comprising individual

FIGURE 1

The median (dorsal, anal, and caudal) and paired (pectoral and pelvic) fins of the bluegill sunfish (*Lepomis macrochirus*).

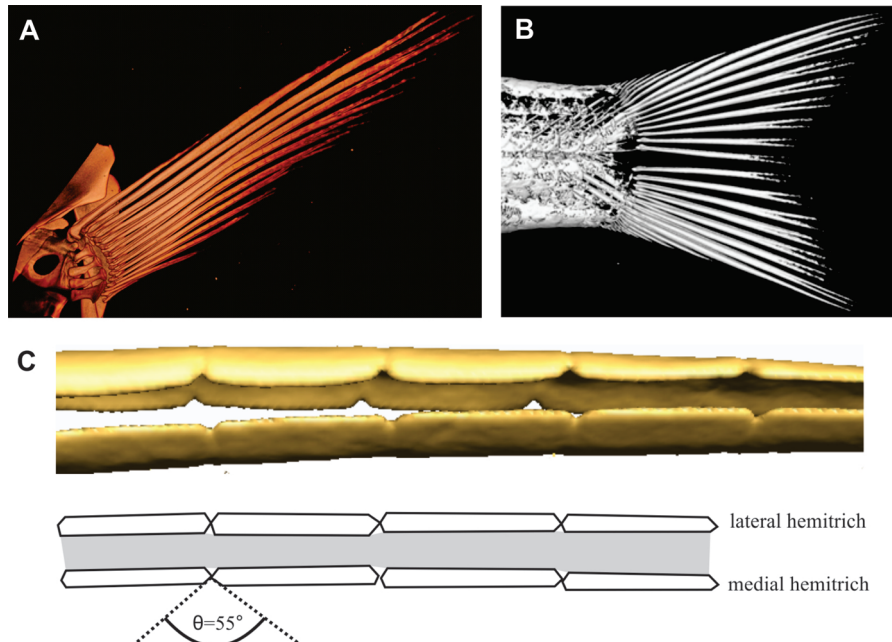


fin rays (lepidotrichia) connected by a thin membrane. Each lepidotrich is controlled by muscles at the base of the fin, such that when one of the halves of a fin ray (hemitrich) slides relative to its other, the fin ray curves significantly (Lauder et al., 2006;

Videler, 1993); yet these extremely flexible fin rays are made of highly mineralized acellular bone (Moss, 1963). Flexible bone is achieved by partitioning the distal 60% of the fin ray into equal-sized segments, each of which can bend approximately 55° rel-

FIGURE 2

Computed microtomography (microCT) scans of (A) left pectoral fin, (B) caudal fin, and (C) individual fin ray (lepidotrich) in sagittal view. Images modified from Esposito et al. (2012) and Flammang et al. (2013).



ative to its neighbor (Figure 2; Alben et al., 2007; Flammang et al., 2013; Geerlink & Videler, 1987).

Having flexible fins confers numerous selective advantages, particularly for locomotion, damping perturbations, and providing for sensory feedback (Hale & Williams, 2012; Williams et al., 2013). Living fishes can modulate the flexibility (or relative stiffness) of their fins and thereby increase thrust production; the same mechanics have been replicated in both robotic and computational models of fish fins (Alben et al., 2007; Flammang, 2010; Lauder et al., 2011b, 2012; Lauder & Madden, 2007; Lauder et al., 2006; Mittal et al., 2006; Tangorra et al., 2011a). Fish actively tune the stiffness of their fins to counteract hydrodynamic loading yet maintain curvatures that preserve thrust production (Flammang et al., 2013; Lauder & Madden, 2007). Modulation of individual fin ray curvature permits precise control of fin shape overall, allowing fish to orient thrust production in specific directions (Flammang & Lauder, 2009, 2016; Lauder & Madden, 2007; Standen & Lauder, 2007).

Flexible fins may offer an advantage in passively damping fluid perturbations that would otherwise disrupt fin function (Flammang et al., 2013). In a series of experiments using a custom-designed vortex generator (Seth et al., 2017), we found that vortex perturbation of the pectoral fins of swimming fish, while causing major deformation to the fin shape at point of impact, had very little effect on more proximal portions of the fin ray driving the fin motion (Flammang et al., 2013). Vortex impact with the fin induced localized buckling between fin ray segments, which were observed as small, focused epicenters of high

curvature along individual fin rays near the point of impact; these curvature rates were several orders of magnitude greater than fin curvature during normal swimming. Notably, deformation of the fin was constrained to the distal, segmented portion of the fin, allowing the proximal portion of the fin to maintain propulsive efficacy.

Lastly, the flexible nature of teleost fin rays aids in interaction with the surrounding environment. Many bony, flexible-finned fishes live in complex environments that require morphological specializations for maneuvering and habitat usage (Crowder & Cooper, 1982; Ellerby & Gerry, 2011; Flammang, 2014). Fishes can derive sensory information from these habitats through the curvature of their flexible fin rays; blind cavefish follow walls by tapping with their pectoral fins to determine their position from the wall (Baker & Montgomery, 1999; Patton et al., 2010). In addition, bluegill sunfish have been shown to increase the frequency of fin tapping while navigating an obstacle course under sensory deprivation conditions (Flammang & Lauder, 2013). Fin ray bending as a result of contact with external surfaces produces sensory information that enhances navigation of complex environments (Hale & Williams, 2012; Williams et al., 2013), which could be useful in programming feedback control algorithms for robotic fishes.

Multifin Coordination of Swimming

To swim and maneuver in a cluttered, three-dimensional fluid environment, fishes have multiple flexible fins that can each be shaped into complex conformations through fine motor control (Flammang & Lauder,

2009; Lauder & Madden, 2007; Lauder et al., 2006; Tytell, 2006). Importantly, all of these fins, which are constantly being modulated in shape, must act in a coordinated effort in order to stabilize rotation about the center of mass (COM) of the fish (Flammang & Lauder, 2016). This is especially true for maneuvering behaviors, such as in backward swimming, because the COM in many fish is not in the geometric center of the fish (Drucker & Lauder, 1999; Tytell & Lauder, 2008); instead, the geometric center is the center of buoyancy. Also, the center of pressure from fin forces is not at the COM; therefore, torques generated by fins with respect to the leading edge differ depending on the direction of motion. The kinematic differences observed between forward and backward swimming can be attributed to controlling roll, pitch, and yaw instability with respect to a COM that is in a different position relative to the direction of travel (Flammang & Lauder, 2016). As a result, in backward swimming, pectoral fin rowing appears to involve more drag-based dynamics as the fin surface is relatively flat during (reverse) thrust generation, whereas in forward swimming, modulation of pectoral fin shape results in lift-based forces and complex vortex generation mechanisms (Lauder & Madden, 2007). Consideration of fin shape modulation, with respect to its effects on the position of the fish in space, could augment passive self-stabilization in robotic swimming fish if implemented properly.

Fishes coordinate fin motion in numerous ways, depending on the speed and style of swimming behavior performed. At slow speeds, including forward swimming and hovering, bluegill sunfish primarily use their pectoral fins, in symmetrical or

alternating patterns, respectively (Flammang et al., 2013; Gibb et al., 1994; Lauder & Madden, 2007; Lauder et al., 2006). Swimming faster than 0.5 body lengths per second tends to involve more caudal fin control of thrust production (Flammang & Lauder, 2008; Gibb et al., 1994; Tytell, 2006), and as the fin with the largest surface area, modulation of caudal fin shape is very important for executing rapid acceleration, braking, or backward swimming behaviors (Flammang & Lauder, 2009, 2016). While long underestimated in their propulsive roles, the dorsal and anal fins of fishes have also been shown to produce substantial thrust, both in forward and backward swimming, as well as help counteract roll instability (Flammang & Lauder, 2016; Standen & Lauder, 2005, 2007; Tytell et al., 2008). During forward swimming, vortices produced by the dorsal and anal fins are entrained with and augment the vortex produced by the caudal fin, resulting in increased thrust production (Flammang et al., 2011; Tytell, 2006); thrust production by wake interaction of the dorsal and anal fins with the caudal fin is dependent on the timing of vortex shedding and distance between the fins (Flammang et al., 2011; Standen & Lauder, 2007).

Design of a Bioinspired Fish Robot

Because the functional morphology and biomechanical properties of fish fins underlie their swimming performance, the hierarchical organization of fish fins, including flexible fin rays and multiple coordinated fins, was included to take advantage of their hydrodynamic properties and

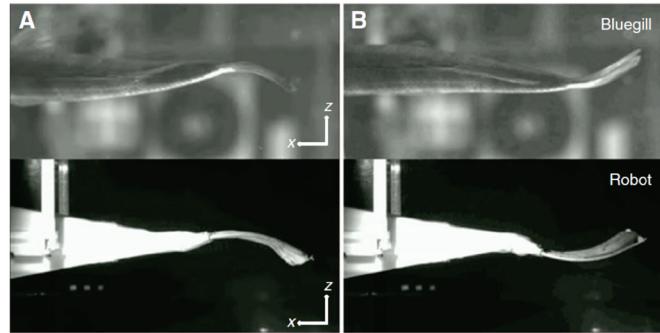
provide a foundation for programming control algorithms.

Each of the fins (pectoral, dorsal, anal, and caudal) was controlled by individually actuated fin rays covered in a polyester-elastane (84%–16%) webbing. The pelvic fins are treated as passive fins in the current robotic model, as our studies on live bluegill showed that they are not used in the majority of locomotor behaviors. The fins were scaled to four times in all linear dimensions relative to an average-sized adult bluegill sunfish (approximately 200 mm total length). However, as compared to live fish, a minimum number of fin rays in each fin were used to reduce design complexity and potential for failure, the number and position of which were determined by proper execution of kinematic repertoires. Using experimentally derived flexural rigidities for bluegill fin rays (Alben et al., 2007; Esposito et al., 2012; Lauder et al., 2011a) and computed microtomography scans to estimate the area moment of inertia along the length of the ray, robotic fin rays were scaled to bend similar to biological fin rays under hydrodynamic loading (Figures 1 and 3).

The pectoral fin was first designed and studied as an independent unit before integration into the whole robot (Figure 4A); this permitted optimization of the motor program without interference of fin-fin interactions and their resulting instability (Phelan et al., 2010; Tangorra et al., 2010, 2008). The complex motions of the pectoral fin were decomposed using proper orthogonal decomposition (POD) into four representative orthogonal modes (Bozkurttas et al., 2009), which were investigated using the robotic model with fin rays of varying stiffness. It was determined that the cupping and sweep mo-

FIGURE 3

Images from high-speed video to show the comparison between the kinematics of the bluegill sunfish caudal fin (top) and that of the robotic caudal fin (bottom). Two representative times are shown 180° out of phase to illustrate caudal fin deformation during locomotion. The robotic caudal fin is driven with a stiff body, in contrast to the deforming bluegill body, and robotic fin rays are actuated at joints with the caudal peduncle. Images have been scaled to the same approximate size; the actual length of the robotic tail is four times that of the bluegill tail. Similar curvatures between the robotic caudal fin and the bluegill tail are achieved with a scaled fin ray stiffness of 150× shown here. Figure modified from Esposito et al. (2012).

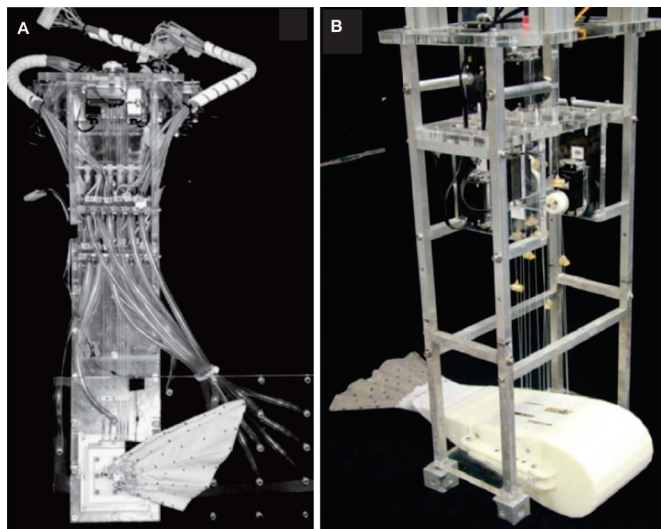


tions of the fin, in addition to the passive flexibility of the fin rays, were important to produce positive thrust throughout the fin beat (Tangorra

et al., 2010, 2008). The pectoral fin was instrumented with a distributed sensory system of static pressure sensors and strain gage sensors to measure

FIGURE 4

Bioinspired robotic pectoral and caudal fin systems. (A) Lateral view of pectoral fin, flat plate with pressure ports behind the fin, and supporting framework to guide low-stretch string tendons to the fin ray base. (B) Caudal fin robot and its frame that houses the rotational servomotors, controller board, and attachment mechanisms for the carriage system. The tendons, attached to the servomotor horns, are guided through the body to the fin rays. Black markers on both fins allow for video tracking of kinematics. Figure modified from Phelan et al. (2010) and Esposito et al. (2012).



the curvature of the fins and provide feedback on hydrodynamic loading and proprioception similar to the innervation in live fish fins (Phelan et al., 2010; Williams et al., 2013). Importantly, the pectoral fin was validated in its ability to create the same kinematic motions and hydrodynamic forces as a bluegill fin, which allowed it then to be used to explore the effect of fin stiffness on variation in fin beat frequency and flow speed (Tangorra et al., 2010).

The caudal fin was also designed and studied independently before being integrated into the full fish robot (Figure 4B). As with the pectoral fin, POD was used to derive five movement patterns from three-dimensional kinematic data of the caudal fin during live fish swimming (Flammang & Lauder, 2008, 2009). The caudal fin robot was also validated using kinematic analysis and digital particle image velocimetry (DPIV; Figure 5) allowing for further comparison of the effect of fin stiffness and motion program (Esposito et al., 2012). These experiments were pivotal in determining that it was the undulation of the caudal fin—and not the lateral translation—that maximized both the thrust and lift generation during steady swimming (Esposito et al., 2012). It was also determined that fin rays in an intermediate stiffness range produced the largest thrust and largest lift; this is consistent with previous experiments that indicated an optimal stiffness in flapping foil motion (Lauder et al., 2011a; Lauder & Madden, 2006).

The body was modeled after the bluegill sunfish (Figure 6) and made of five water-tight modular sections: head, pectoral, middle, tail, and peduncle. Modularity allowed for discrete motor compartmentalization

FIGURE 5

DPIV images, from a lateral view, of (A) a bluegill sunfish and (B) the robotic caudal fin during rolling tail motion. The fish caudal fin and the robotic caudal fin have been superimposed to show their position relative to the flow. In the flow images, yellow/red represents a positive (counter-clockwise) rotation of fluid, whereas blue/purple represents a negative (clockwise) rotation of fluid. Yellow and black arrows indicate the velocity and direction of the fluid. For both images, the green regions indicate an area of zero vorticity. The flow is moving in the positive x -direction. Figure modified from Esposito et al. (2012).

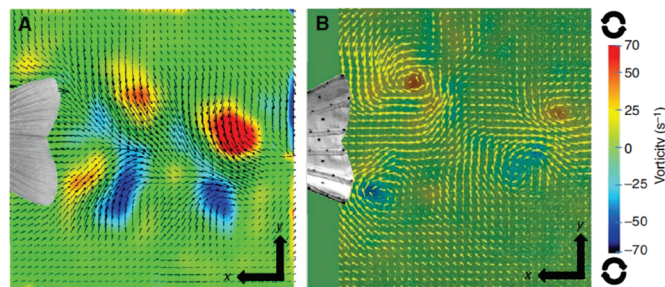
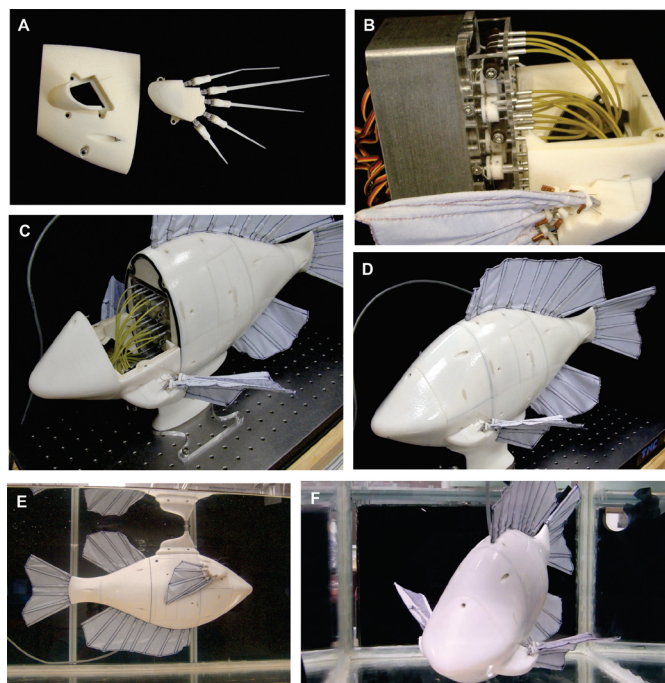


FIGURE 6

Design, construction, and testing of the autonomous bioinspired fish robot. Multiple pectoral fin configurations can be explored by using fin inserts with different geometries and numbers and sizes of fin rays (A). Each pectoral fin is attached to the body via the fin's mounting plate. Each fin ray is driven using a pair of sheathed cables (B) that are actuated using a single servomotor. The 10 servo motors used to actuate both fins are enclosed in a waterproof aluminum box, which also encloses a microcontroller and motor driver dedicated to each pair of fins (B). The microcontroller can be programmed for different control strategies, ranging from open-loop, planned trajectories to sensory-mediated neural oscillators. The use of cable-driven fin rays enables the servomotors and controllers to be positioned within the robot's body away from the fin in a manner that creates a desirable location for the robot's COM (C and D). The robot can be tested while attached to an instrumented sting (E) or while swimming freely (F) with power coming via a tether.



and the ability to change a single section for future designs. The dorsal and anal fins are attached in the tail section, which is currently rigid, but could be made of a flexible material and actuated to undulate like a biological fish. The tail portion of the body tapers smoothly toward the caudal peduncle to reduce total drag and minimize the turbulent effects of altered water flow over the robotic tail. Pressure sensors were placed on the body surface to replicate the lateral line of fishes and produce information on surrounding fluid flow and fin-body interactions (Tangorra et al., 2011b). An inertial measurement unit (ArduIMU V3) was placed in the head region to model the vestibular sense and measure changes in body orientation.

A central pattern generator (CPG) model was chosen to control the motion of the fish for swimming for several reasons, including that many biological fish motor patterns are driven by CPGs and such models have already proven successful in other robotic applications (Zhou & Low, 2010). The CPG was used to drive fin motions for steady swimming, but body orientation was driven using the ArduIMU in a PD controller configuration. The ArduIMU also provided descending inputs that affected feedback configuration and gain settings. The Matsuoka neural oscillator model was selected for the CPG because it could generate approximations of the biological swimming gaits, was biologically relevant allowing knowledge acquired about the architecture of fish CPGs to be incorporated into the existing mathematical framework, could change gait in response to environmental and supervisory feedback, and could be implemented on a microcontroller (Tangorra et al., 2011b). The oscillator produced reason-

able approximations of the steady swimming and undulation motions exhibited by the sunfish. Ongoing work is focused on developing multi-neuron networks that incorporate bending sensing from fins and descending inputs from the ArduIMU and on understanding how the architecture of the feedback (inhibitory and excitatory) and the introduction of interneurons affect the ability of CPG to create multiple gaits and to entrain the CPGs with fin resonance characteristics.

Conclusions

Fin stiffness—and more specifically, the ability to actively modulate stiffness—is key to maximizing thrust forces. Using validated, bioinspired robotic fish fins, we were able to tease apart some aspects of the complicated relationship between fin stiffness, fin shape, flapping frequency, and flow rate. These studies of individual fins provide the basis for the design of a complete fish-like robotic system that will be the subject of future testing. Interestingly, maximum forces at different points in the fin beat came from fins of different stiffness and different shapes. Therefore, there is not a simple relation between fin stiffness and shape and fin force. Fish fins are dynamic structures coupled with the fluid environment, and both the fin and fluid need to be considered together in robotic design. To maximize thrust and the impulse imparted to the water, it will likely be necessary to alter the fin's stiffness and its dynamic interaction with the water throughout the fin beat and as operating conditions change.

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